XC-algebras and quantum knot invariants

Jorge Becerra

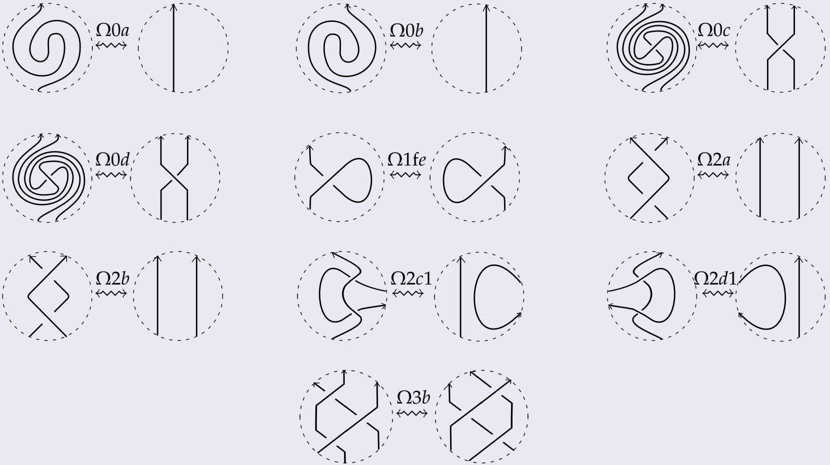
Université Bourgogne Europe

Topology Seminar UNIGE
28 May 2025

Slides available at `bit.ly/jbecerra`

Theorem (B.-van Helden 2025)

The following is a (non-minimal) generating set of rotational Reidemeister moves for rotational knot diagrams:



XC-algebras

Definition

Let A be a $\Bbbk$-algebra. An *XC-structure* on A is the choice of two invertible elements

$$R \in A \otimes A \qquad , \qquad \kappa \in A$$

satisfying

(XC0) $R^{\pm 1} = (\kappa \otimes \kappa) \cdot R^{\pm 1} \cdot (\kappa^{-1} \otimes \kappa^{-1})$,

(XC1f) $\sum_i \beta_i \kappa \alpha_i = \sum_i \alpha_i \kappa^{-1} \beta_i$,

(XC2c) $1 \otimes \kappa^{-1} = \sum_{i,j} \alpha_i \bar{\alpha}_j \otimes \bar{\beta}_j \kappa^{-1} \beta_i$,

(XC2d) $\kappa \otimes 1 = \sum_{i,j} \bar{\alpha}_i \kappa \alpha_j \otimes \beta_j \bar{\beta}_i$,

(XC3) $R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}$.

The triple (A, R, κ) is called an *XC-algebra*.