(Not) A nought-level Knot talk

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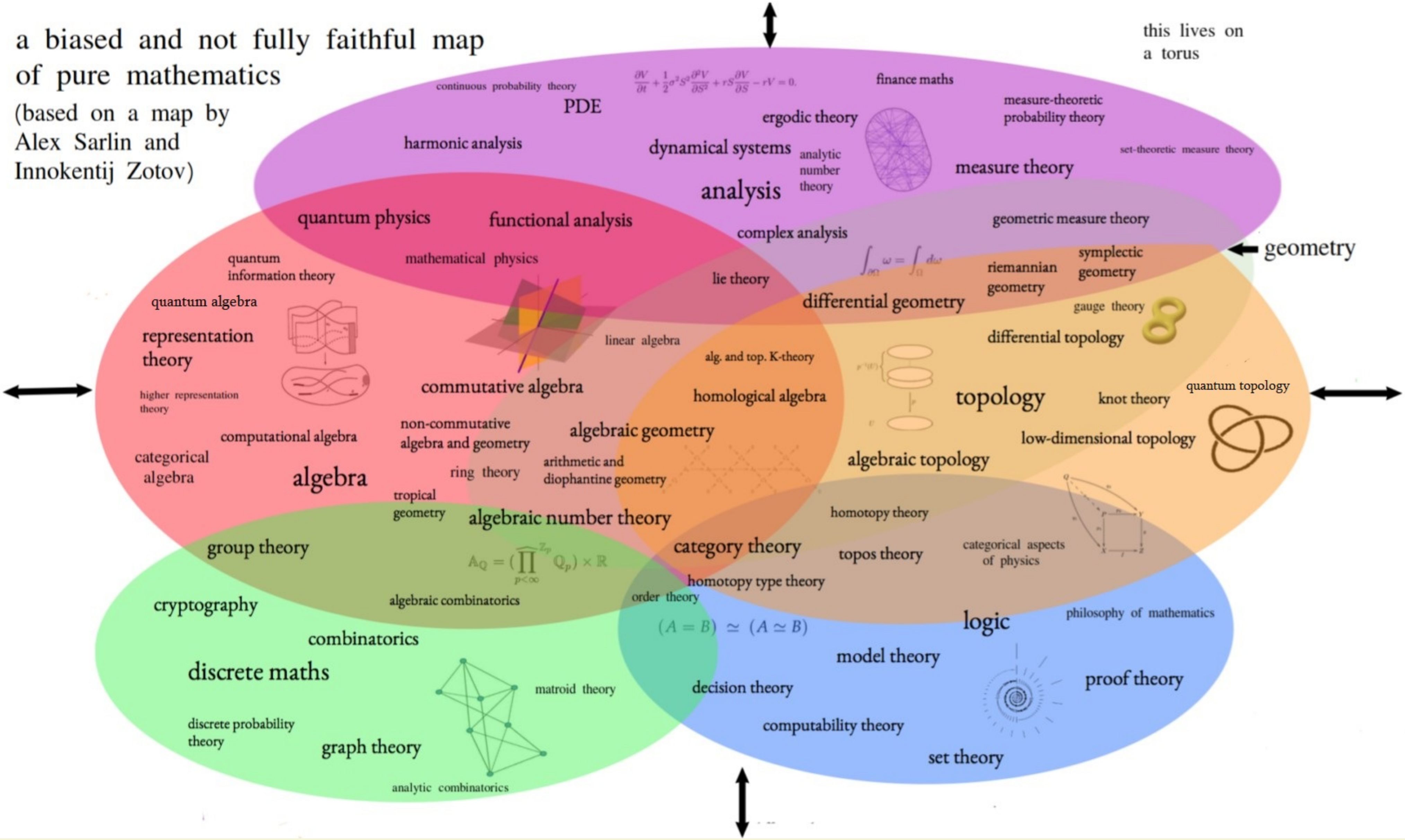
The map of pure mathematics.

Where do I stand in maths?



a biased and not fully faithful map of pure mathematics

(based on a map by Alex Sarlin and Innokentij Zotov)

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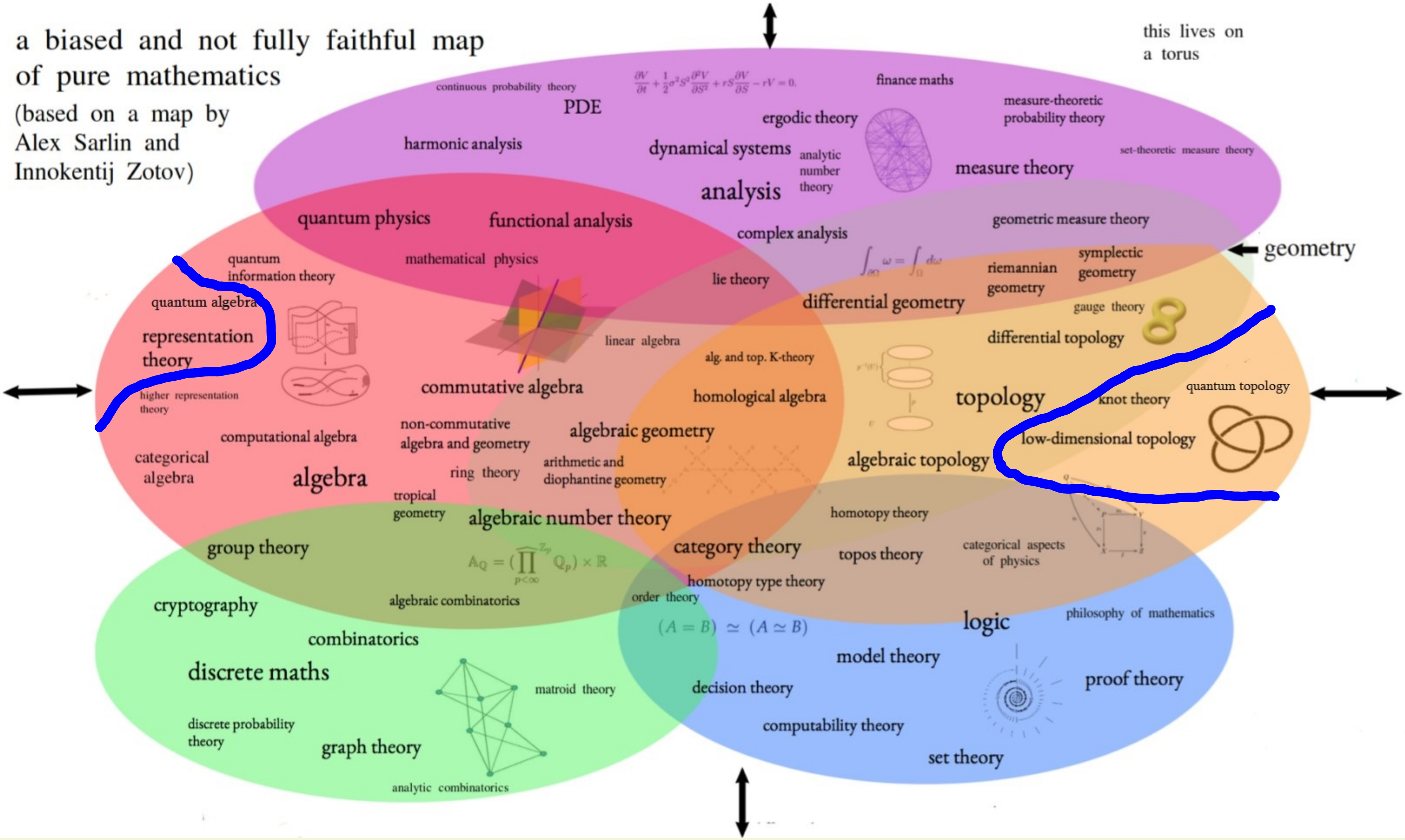

- In algebra $\cap$ topology = algebraic topology, one typically studies algebraic invariants of topological objects:

- topological space
- (smooth) manifold
- embedded manifold (e.g. knots and links)

an algebraic gadget (e.g a number, a group, a polynomial, an elmt of some algebra, ...) which is

- homotopy / topological invariant
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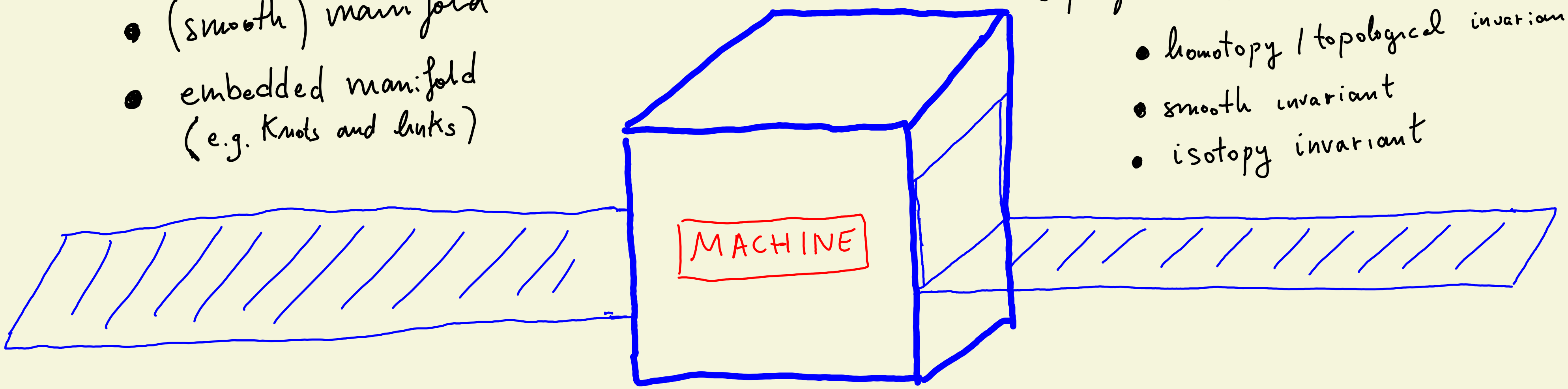
- isotopy invariant



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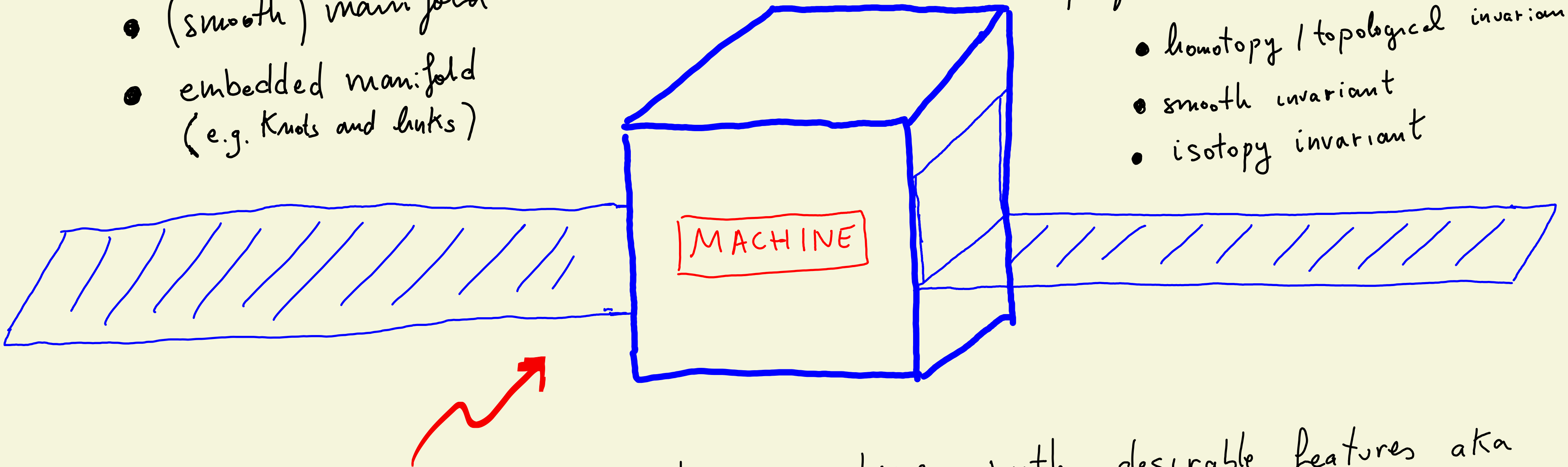

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Usually one is most interested in machines with desirable features aka "nice properties", e.g. functoriality, naturality wrt some construction of the topological object, ...
(e.g. $H_n/\pi_n : \mathrm{Top} \to \mathrm{Grp}$, $\pi_n(X \times Y) \cong \pi_n(X) \times \pi_n(Y)$, Künneth for H_n , . . .)

- In quantum topology ⊂ algebraic topology, the MACHINE usually depends on the choice of some algebraic gadget, e.g

(1) A power series $\varphi \in \mathbb{Q}\langle\langle X, Y \rangle\rangle$,

(2) A monoidal category (w/ extra structure)

(3) An algebra (w/ extra structure)

⋮

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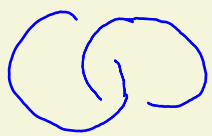
  $\vdots$

- Using these objects one can construct isotopy invariants of knots and links in $\mathbb{R}^3$.



Q : Why should you care about this?

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- There is a construction

$$\left\{ \begin{array}{c} \text{(framed) knots} \\ \text{and links in } \mathbb{R}^3 \end{array} \right\} \xrightarrow{\text{surgery}} \left\{ \begin{array}{c} \text{closed, oriented,} \\ \text{connected} \\ \text{3-manifolds} \end{array} \right\}$$

w/ the property that the invariants (1) - (3) before can be upgraded to 3-mnfd invariants that, in some cases, distinguish homotopy equivalent spaces (so <u>finer</u> than H_n, H^n, π_n, ...)

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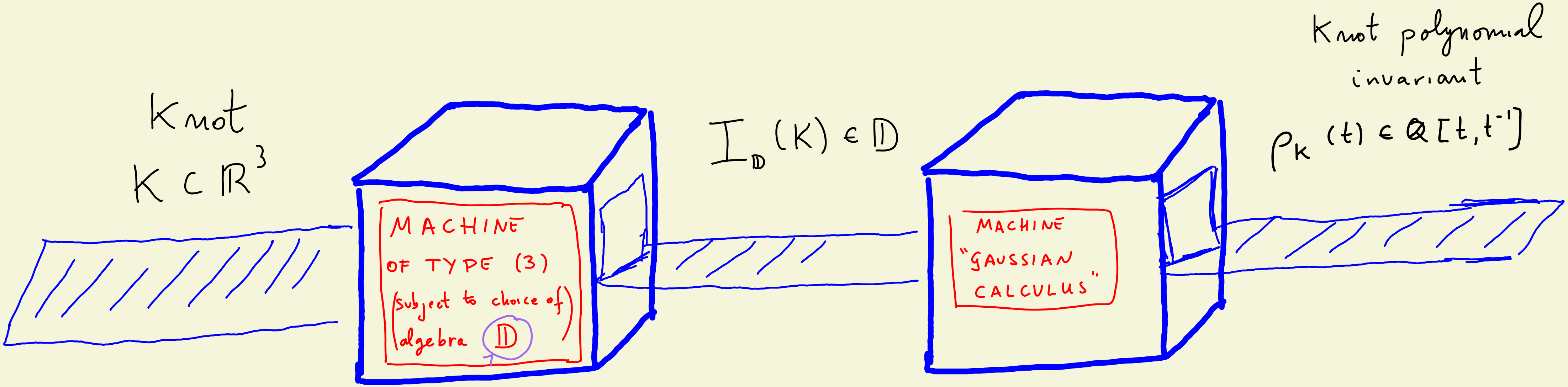

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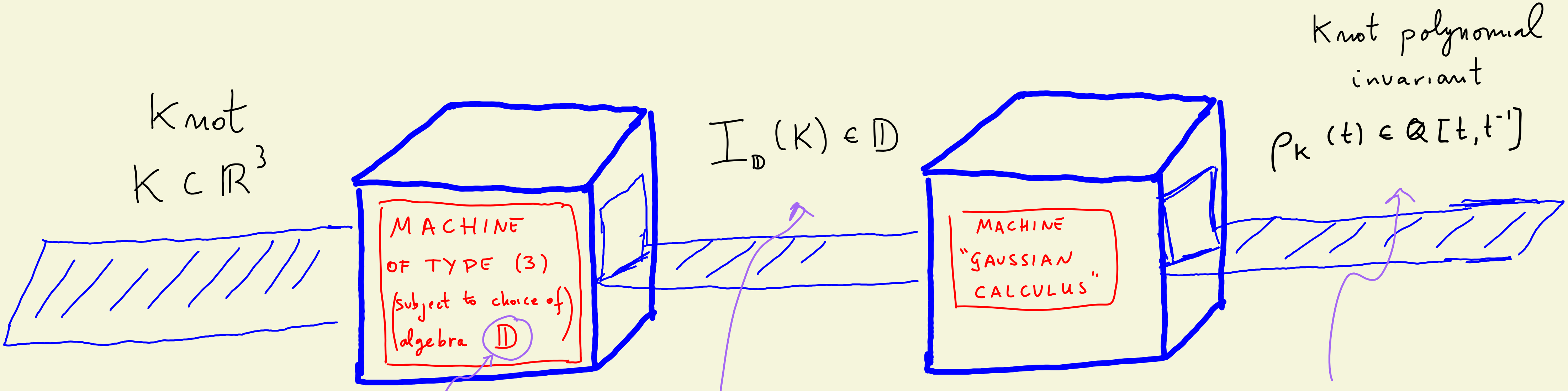

- top algebra / $\mathbb{Q}[[h]]$
- ∞-dim / $\mathbb{Q}[[h]]$
- non-commutative

too difficult!!

(deciding whether $a = b$ in $\mathbb{D}$ is really hard)

very easy to decide if two are or not equal!

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Theorem (Paraphrase). If a knot K bounds $(S^1 \times S^1) - \mathring{D}^2$ in $\mathbb{R}^3$ then ρ_K is fully determined by 7 integers

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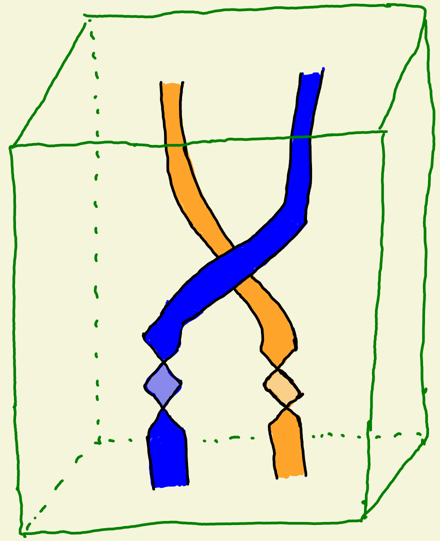

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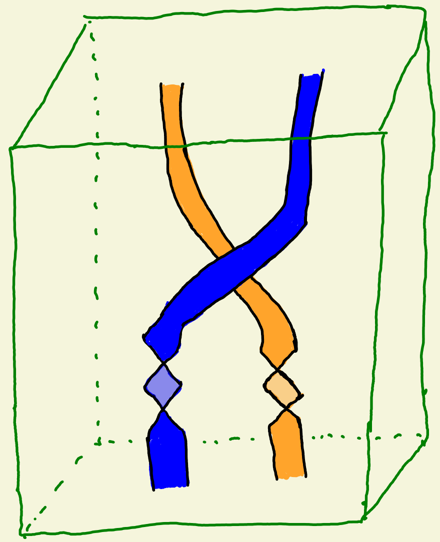

Corollary (Paraphrase). If K is as above, then ρ_K (which can be obtained by computer, and fast) with another knot polynomial invariant Θ_K (hard to compute).