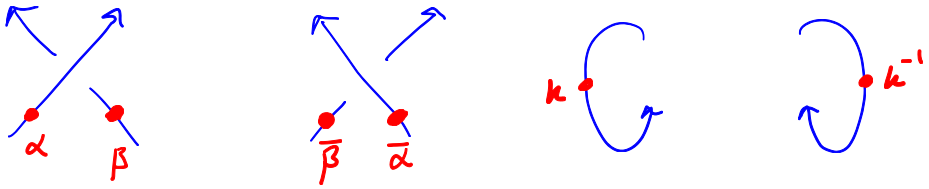
Different conventions for the universal quantum invariant

[BNV] :

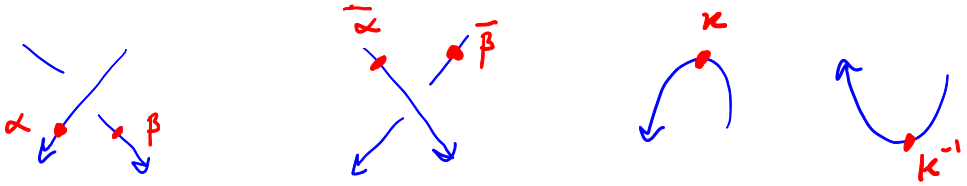
Bar-Natan van der Veen



writing (from left to right) following the order given by the orientation.

[Ohtsuki]

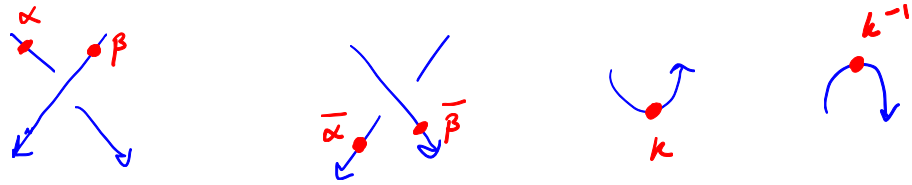
Ohtsuki



writing (from left to right) following the order given by the orientation.

[Habiro]

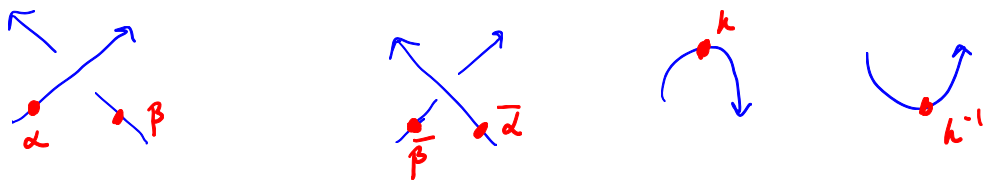
Habiro



writing (from left to right) following the order opposite to the orientation.

[BBG]

Beliakova Blanchet Geer



writing (from left to right) following the order opposite to the orientation.

- They relate as follows, for a tangle T:

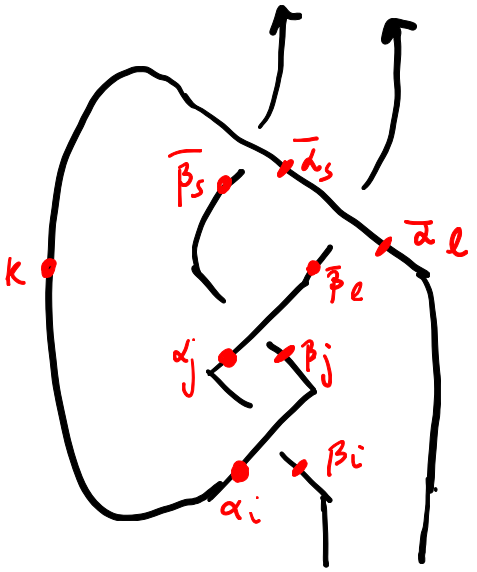
$$Z_A^{BNV}(T) = Z_A^{Oht}\left(\begin{array}{c}T \text{ rotated } 180^\circ \text{ on the}\\ \text{plane, ie upside down}\end{array}\right).$$

$$Z_A^{BNV}(T) = Z_A^{Hab}\left(\begin{array}{c}T \text{ rotated } 180^\circ \text{ along the } z\\ \text{axis and with opposite}\\ \text{orientation on strands}\end{array}\right)$$

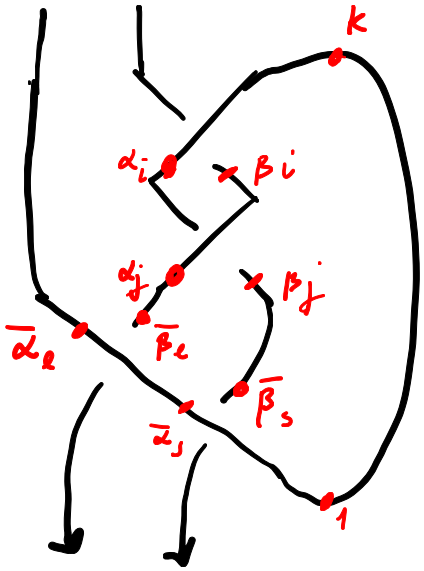
$$Z_A^{BNV}(T) = Z_A^{BBG}\left(\begin{array}{c}T \text{ rotated } 180^\circ \text{ on the}\\ \text{plane and with opposite}\\ \text{orientation on strands}\end{array}\right)$$

Example: For the following tangles, their invariants according to the conventions BNV, Oht, Hab, BBG are all equal to

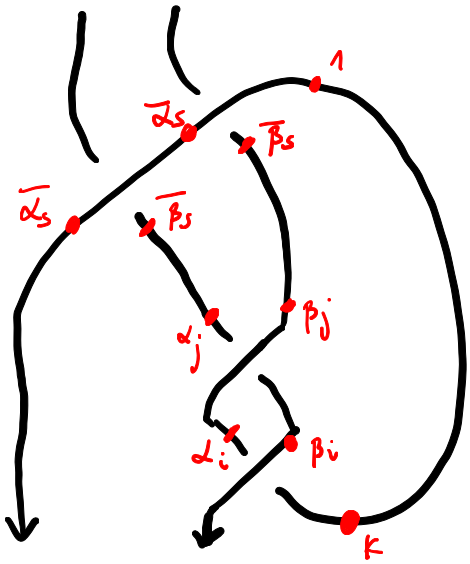
$$(*) := \sum_{i,j,k,s} \beta_i \alpha_j \bar{\beta}_k \otimes \bar{\alpha}_k \bar{\alpha}_s \kappa \alpha_i \beta_j \bar{\beta}_s.$$



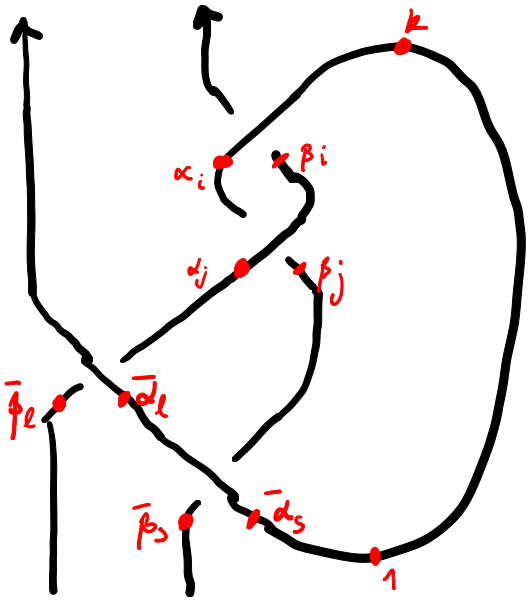
$Z_A^{BNV} = (*)$



$Z_A^{Oht} = (*)$



$Z_A^{Hab} = (*)$



$Z_A^{BBG} = (*)$