

## Proof of Lemma 3.8:

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In[*]:= (*We establish the multiplication for SW. We
set the variable l to be sw. The bar denotes inverse. *)
vars = {s, w, l};
mi,j→k[Z_] := Expand[Z] /. {si sj → 1, wi wj → 0, si wj → lk, wi sj → -lk, li sj → -wk, si lj → wk,
li wj → 0, wi lj → 0, li lj → 0, x-i /; MemberQ[vars, x] → xk, x-j /; MemberQ[vars, x] → xk}}
```

(\*We start by x\*)

```
In[*]:= xi- := λ1 + λ2 si + λ3 wi + λ4 li;
x̄i- := λ̄1 + λ̄2 si + λ̄3 wi + λ̄4 li;
p = (x1 x̄2 // m1,2→1) - 1;
vari = {s1, w1, l1};
cr = CoefficientRules[p, vari];
eqs = Thread[Values[cr] == 0];
(*Computing the determinant of the matrix of the linear system*)
CoefficientArrays[eqs, {λ̄1, λ̄2, λ̄3, λ̄4}] [[2]] // Normal // Det // Simplify
```

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Out[*]:=
-(λ12 - λ22)2
```

```
In[*]:= (*Solving the system*)
Solve[eqs, {λ̄1, λ̄2, λ̄3, λ̄4}]
```

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Out[*]:=
{{λ̄1 → λ1 / (λ12 - λ22), λ̄2 → λ2 / (-λ12 + λ22), λ̄3 → -λ3 / (λ12 - λ22), λ̄4 → -λ4 / (λ12 - λ22)}}
```

```
In[*]:= (*Now similarly for Q*)
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```
Qi,j- := μ1,1 + μ1,2 si + μ1,3 wi + μ1,4 li + μ2,1 sj + μ2,2 si sj + μ2,3 wi sj + μ2,4 li sj +
μ3,1 wj + μ3,2 si wj + μ3,3 wi wj + μ3,4 li wj + μ4,1 lj + μ4,2 si lj + μ4,3 wi lj + μ4,4 li lj;
Q̄i,j- := μ̄1,1 + μ̄1,2 si + μ̄1,3 wi + μ̄1,4 li + μ̄2,1 sj + μ̄2,2 si sj + μ̄2,3 wi sj + μ̄2,4 li sj +
μ̄3,1 wj + μ̄3,2 si wj + μ̄3,3 wi wj + μ̄3,4 li wj + μ̄4,1 lj + μ̄4,2 si lj + μ̄4,3 wi lj + μ̄4,4 li lj;
```

```
vari = {s1, s2, w1, w2, l1, l2};
p = (Q̄1,2 Q3,4 // m1,3→1 // m2,4→2) - 1;
cr = CoefficientRules[p, vari];
eqs = Thread[Values[cr] == 0];
(*Computing the determinant of the matrix of the linear system*)
CoefficientArrays[eqs, {μ̄1,1, μ̄1,2, μ̄1,3, μ̄1,4, μ̄2,1, μ̄2,2, μ̄2,3,
μ̄2,4, μ̄3,1, μ̄3,2, μ̄3,3, μ̄3,4, μ̄4,1, μ̄4,2, μ̄4,3, μ̄4,4}] [[2]] // Normal // Det // Factor
```

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Out[*]:=
-(μ1,1 + μ1,2 - μ2,1 - μ2,2)4 (μ1,1 - μ1,2 + μ2,1 - μ2,2)4 (μ1,1 - μ1,2 - μ2,1 + μ2,2)4 (μ1,1 + μ1,2 + μ2,1 + μ2,2)4
```

```
In[*]:= (*Solving the system*)
```

```
Solve[eqs,
{μ̄1,1, μ̄1,2, μ̄1,3, μ̄1,4, μ̄2,1, μ̄2,2, μ̄2,3, μ̄2,4, μ̄3,1, μ̄3,2, μ̄3,3, μ̄3,4, μ̄4,1, μ̄4,2, μ̄4,3, μ̄4,4}]
```

[illegible]