COURSE SYLLABUS FOR TOPICS IN TOPOLOGY 2023

Time. Tuesdays and Thursdays 3pm-5pm

Location. NB 5118.0161 Tuesdays | U-building 203 Thursdays

Instructors.

- Roland van der Veen, office 468, r.i.van.der.veen@rug.nl
- Jorge Becerra, office 493 BB, j.becerra@rug.nl

Office hours. By appointment.

Course website. sites.google.com/view/becerra/TT23

Text. Course notes will be provided on the course website.

Prerequisites. Introduction to metric and topological spaces, linear algebra, group theory, multivariable analysis. Analysis on manifolds and, advanced algebraic structures are recommended but not required.

Content. We will give an introduction to algebraic methods in knot theory and low-dimensional topology.

Take a piece of string and tie a knot. Can you tell what kind of knot it is just by looking at it? What are its properties? We will turn any picture of a knot into a computation in such a way that the final answer only depends on the knot itself. Such a computation is called a knot invariant. Knot invariants are central to low-dimensional topology and serve to connect this field with other areas of mathematics in unexpected ways.

Guided by the close analogies between pictures of knots and the representation theory of algebras we will provide a general framework for knot invariants. Famous examples such as the Jones polynomial and the Alexander polynomial come up naturally this way. The relevant algebras turn out to be Hopf algebras with close ties to the theory of Lie groups and quantum physics. Dealing with such algebras directly is challenging but can done either through studying their representations or by turning linear maps into power series.

Grading. Grades will be based on homework (65%), a presentation at the end of the course (25%) and practical work (10%).

Homework will be assigned biweekly on Thursdays and due before the Thursday lecture of the following week. Students will work in their homework individually. You are also expected to write down full sentences in English, so instead of "$X \subset \mathbb{R}$ closed & bounded $\Longrightarrow$ X compact" you should write "Since $X \subset \mathbb{R}$ is closed and bounded, it is also compact". LaTeX-ed solutions are preferred, but you are also allowed to present handwritten solutions as long as they are written clearly.

The presentations will take place on 28 March, 30 March and 3 April. A list with the possible topics will be available no later than 16 March. You will be expected to prepare a 30-minute presentation and a detailed LaTeX-ed handout to be brought

to your presentation. Handouts completed after the presentation will not be taken into consideration.

Practical work will consist of active participation in the course by regular attendance, asking questions and making remarks in the discussions, and also suggesting additional material and solving exercises in class.

Further references.

- Ohtsuki, T. , *Quantum Invariants: A Study of Knots, 3-manifolds, and Their Sets*
- D.M. Jackson, I. Moffat, *An Introduction to Quantum and Vassiliev Knot Invariants*