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Choice of topics 2 - Topics in Topology

It is time to choose your second talk for the course! As last time choose **one** of the following topics and send your choice **before 7 March**to `j.becerra@rug.nl`.

OP=OP! As soon as a presentation is chosen, it will appear on the website schedule, so check this before sending your choice.

List of talks

1. **Planar Heegaard diagram** (10 March)

   Determine the 3-manifold given by the planar Heegaard diagram shown in Figure 1.

   References: This exercise sheet and this document by Marc Kegel.

2. **Heegaard genus of T^3** (10 March)

   Describe a Heegaard splitting of genus 3 for the 3-torus $T^3 = S^1 \times S^1 \times S^1$. State that if a closed, orientable 3-manifold M admits a genus g Heegaard diagram then its fundamental group $\pi_1(M)$ admits a presentation with g generators and g relations (show this claim at least in the notes). Conclude that the Heegaard genus of T^3 is 3.

   References: Jesse Johnson's notes on Heegaard splittings Example 3.13. The claim about the fundamental group is an application of the van Kampen theorem, see those notes, or Rolfsen §9.C, or Saveliev §1.8.

3. **$L(p,1)$ as the boundary of a 4-manifold** (17 March)

   Describe a 4-manifold E_p whose boundary is $L(p,1)$. Show that for $p = 0, 1, -1$ this 4-manifold is precisely $S^2 \times D^2$, $\mathbb{CP}^2 - \overset{\circ}{D}{}^4$ and $(-\mathbb{CP}^2) - \overset{\circ}{D}{}^4$, respectively.

   References: Saveliev pages 40-41.

4. **Torus knots** (24 March)

   Define the torus knot $T_{p,q}$ for (positive) coprime integers p, q and compute the fundamental group of its complement $\pi_1(S^3 - T_{p,q}) \cong \langle x, y | x^p = y^q \rangle$.

   References: Hatcher 1.24; Rolfsen §3.B-C.

5. **Equivalent surgery descriptions I** (24 March)

   Use Kirby calculus to show that the Poincaré homology sphere arises as surgery on the framed link shown in Figure 2.

   References: Saveliev §3.2

6. **Equivalent surgery descriptions II** (31 March)

   Use Kirby calculus to show that the manifold described by the framed link represented by the graph shown in Figure 3 is diffeomorphic to the manifold obtained by 1-surgery on the twist knot of type $2m + 2$.

   References: Saveliev §3.2

7. **Equivalent Kirby diagrams** (31 March)

   Show that the Kirby diagrams (for 4-manifolds) showed in Figure 4 represent the same 4-manifold.

   References: This exercise sheet and this document by Marc Kegel.

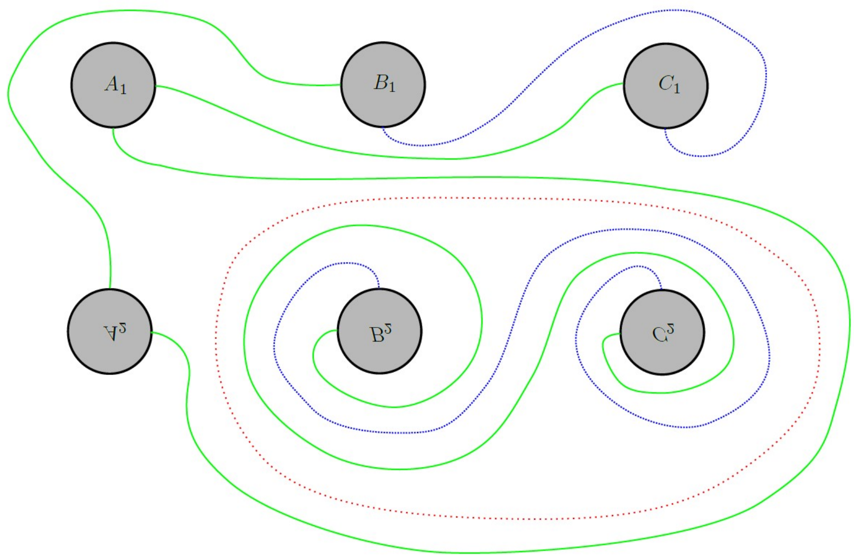

Figure 1: A planar Heegaard diagram.

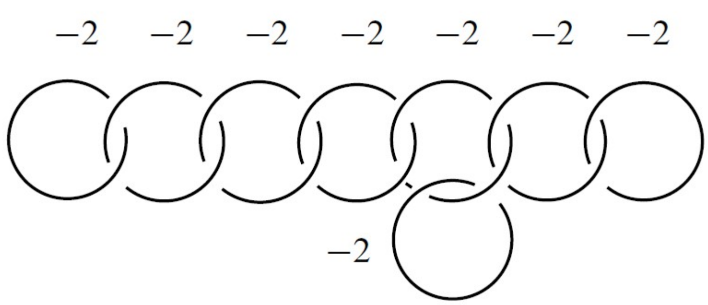

Figure 2: A framed link giving rise to the Poincaré sphere.

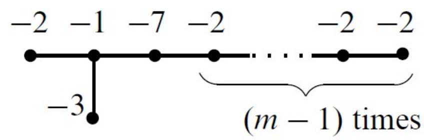

Figure 3: A graph representing a framed link.

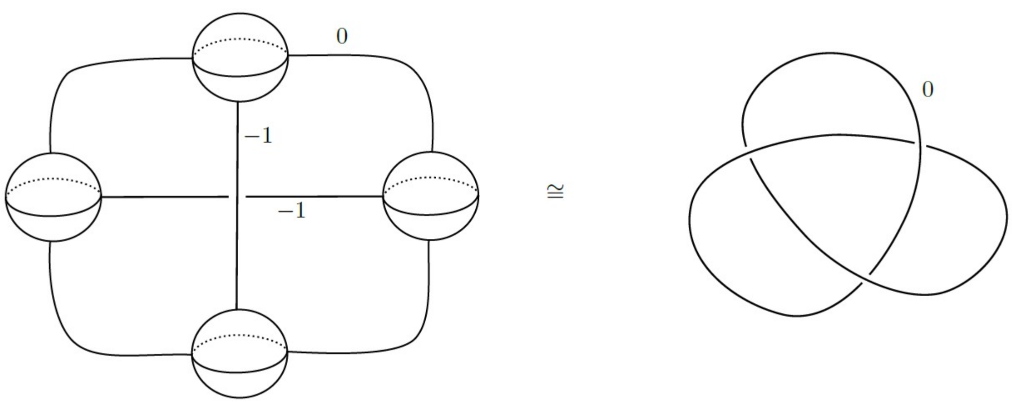

Figure 4: Two Kirby diagrams for the same 4-manifold.