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Choice of topics 1 - Topics in Topology

All students have to give two 30-minute talks in this course. To accompany the talk, a detailed handout must be elaborated. Here we assign the first talk to everyone. You must choose **one** of the following topics and send your choice to `j.becerra@rug.nl`. OP=OP !

List of talks

1. **Homology*** (10 February)

   CW complexes. Define homology for CW-complexes (that is, *define* homology as cellular homology, not as singular homology). Long exact sequence.

   References: Hatcher or the Mastermath course lecture notes, or §5.6 - 5.7 of Turaev's *Introduction to combinatorial torsions* for this direct approach.

2. **Homotopy groups*** (10 February)

   Definition, group structure. The van Kampen theorem. Relation between π_1 and H_1. The homotopy groups of spheres $\pi_k(S^n)$ for $k \leq n$.

   References: Hatcher or the Mastermath course lecture notes.

3. **Classification of closed, connected, orientable surfaces I** (17 February)

   Use handle decomposition to show that the only closed, connected 1-manifold is S^1. Describe handle decompositions for S^2 and T^2 with one 0-handle and one 2-handle. Describe a handle decomposition for the connected sum of surfaces. Show that any handle decomposition of a surface can be modified so that all 1-handles are attached to the 0-handle.

   References: This exercise sheet and this document by Marc Kegel.

4. **Classification of closed, connected, orientable surfaces II** (17 February)

   Kirby diagrams for surfaces. Show how they uniquely determine a surface. Exhibit Kirby diagrams for S^2 and $\#_k T^2$. Show how a Kirby diagram changes through a handle slide. Show that any closed, connected, orientable surface is S^2 or $\#_k T^2$ by transforming the Kirby diagram of an arbitrary surface to a Kirby diagram for S^2 or $\#_k T^2$.

   References: This exercise sheet and this document by Marc Kegel.

5. **Handle decomposition of $\mathbb{RP}^n$ and $\mathbb{CP}^n$** (24 February)

   Describe a handle decomposition of real and complex projective spaces. Explicitly compute the attaching maps and draw the handle decompositions for $\mathbb{RP}^1$, $\mathbb{CP}^1$, $\mathbb{RP}^2$ and $\mathbb{RP}^3$.

   References: Gompf-Stipsicz 4.2.4 and 4.2.5

6. **The lens spaces** $L(p, q)$ (24 February)

   Introduce the lens space $L(p, q)$ as a quotient of S^3. Describe this space as a $(-p/q)$-surgery on the unknot (and show that both descriptions are equivalent). Compute its fundamental group and its homology groups.

   References: Rolfsen §9.B, or Gompf-Stipsicz §5.2

7. **The Poincaré homology 3-sphere I** (3 March)

   Describe the Poincaré homology sphere as (a) the quotient of S^3 modulo the binary icosahedral group and (b) the Brieskorn manifold $\Sigma(2, 3, 5)$, and show that both descriptions are equivalent.

   References: Kirby - Scharlemann's paper, or Rolfsen's book.

8. **The Poincaré homology 3-sphere II** (3 March)

   Describe the Poincaré homology sphere as (a) a manifold obtained from a planar Kirby diagram and (b) as (-1)-surgery on a left-handed trefoil, and show that both descriptions are equivalent.

   References: Kirby - Scharlemann's paper, or Rolfsen's book.