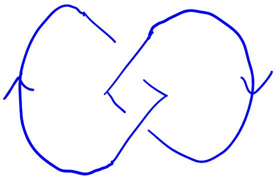
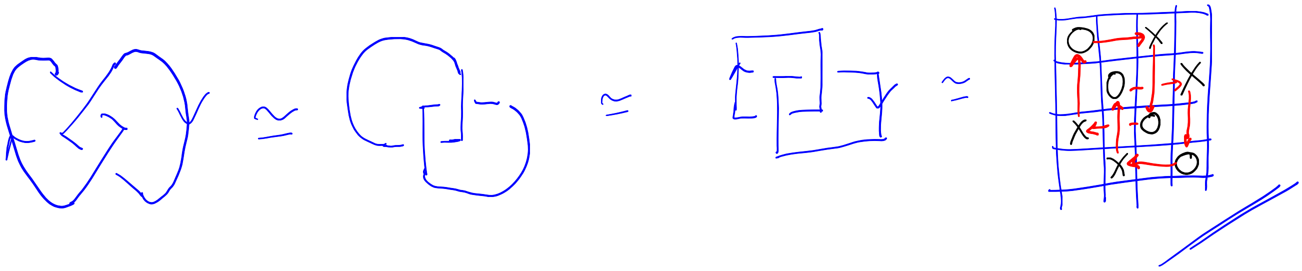
Alexander polynomial of the Hopf link via grid diagrams

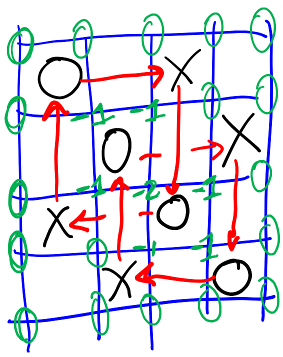
Let $L =$  be the Hopf link.

We can deform the strands to have only vertical and horizontal segments:



grid diagram for Hopf link

The winding numbers are:



Therefore,
$$M(\mathcal{G}) = \begin{pmatrix} 1 & t & t & 1 \\ 1 & t & t^2 & t \\ 1 & 1 & t & t \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

and $\det M(\mathcal{G}) = -(t-1)^4$.

On the other hand,
$$a(\mathcal{G}) = \frac{-1 \; -1 \; -5 \; -1 \; -1 \; -5 \; -1 \; -1}{8} = -2$$

Lastly, $\sigma_\Phi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 4 & 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & \cdots \\ \downarrow & \downarrow & \downarrow & \\ n & n-1 & n-2 & \cdots \end{pmatrix}$ for $n=4$,

so $\varepsilon(\mathcal{G}) = +1$

In total,

$$\Delta_{\text{Hopf link}}(t) = \varepsilon(\mathcal{G}) \cdot \det M(\mathcal{G}) \cdot \left(t^{-1/2} - t^{1/2}\right)^{1-4} \cdot t^{a(\mathcal{G})}$$

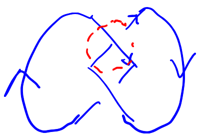
$$= +1 \cdot \left[-(t-1)^4\right] \cdot \left(-t^{3/2}\right)(t-1)^{-3} \cdot t^{-2}$$

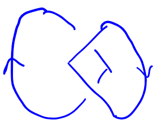
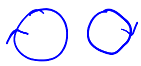
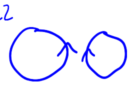
$$= t^{-1/2}(t-1) = \underline{\underline{t^{1/2} - t^{-1/2}}}$$

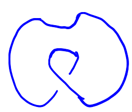
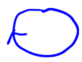
Let's see that we get the same if we use the skein relation

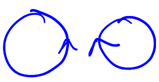
$$\Delta_{L_+} - \Delta_{L_-} = \left(t^{1/2} - t^{-1/2}\right) \Delta_{L_0}$$

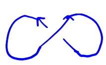
$$\Delta_L - \underbrace{\Delta_{L_-}}_{?} = \left(t^{1/2} - t^{-1/2}\right) \cdot 1$$

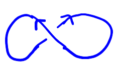
$L =$  $= L_+$

$L_- =$  $\overset{R2}{=}$  $\overset{R1\,\&\,R2}{=}$ 

$L_0 =$  $\overset{R1}{=}$ 

Let $\tilde{L}_0 =$ , then

$L_+ =$  $=$ 

$L_- =$  $=$ 

so $0 = 1 - 1 = \left(t^{1/2} - t^{-1/2}\right)\Delta_{\tilde{L}} \Rightarrow \underline{\Delta_{\tilde{L}} = 0}$

Upshot: $\Delta_{\text{Hopf link}}(t) = \underline{\underline{t^{1/2} - t^{-1/2}}}$, as before.