Knots / isotopy $\cong$ grid diag / comm. & stabilis.

Today (Sven): HG inv under stab.

$\widetilde{GH}$ vs $\mathbb{Z}/2$ (not inv)

$\widehat{GH}$ vs $\mathbb{Z}/2$ (is inv)

GH^- mod over $\mathbb{Z}/2[U]$. $\rightsquigarrow$ $\mathcal{R}[S(\mathbb{G})]$ $\quad V_1 V_2 x$

$CG^- = \mathcal{R}[S(\mathbb{G})]$ $\quad \mathcal{R} = \mathbb{Z}/2[V_1 \dots V_n]$.

$x \in S(\mathbb{G})$. $\rightsquigarrow$ $M(x)$, $A(x)$

Declare V_i to be bideg $(-2,-1)$

$$M(V_1^{k_1} \cdots V_n^{k_n} x) := M(x) - 2k_1 - \dots - 2k_n$$

$$A(\quad) := A(x) - k_1 - \dots - k_1$$

$$CG^- = \bigoplus_{d,s} CG^-_{d,s}$$

C bigr. mod, ∂ has bideg $(-1,0)$

$f: C \to C'$ chain map

$f(C_{d,s}) \subseteq C_{d+m,s+t}$ f of bideg (m,t).

$$0 \to C' \xrightarrow{f} C \xrightarrow{g} \bar{C} \to 0$$

$(-2,-1)$ $(0,0)$

les where f_*, g_* preserve the bigrading & δ has bideg $(1,1)$.

$$\begin{tikzcd} H(C') \arrow[rr, "f_*"] & & H(C) \arrow[dl, "g_*"] \\ & H(\bar{C}) \arrow[ul, "\delta"] & \end{tikzcd}$$

exact

unravel

$$H_{d+2,s+1}(C') \xrightarrow{f_*} H_{d,s}(C) \xrightarrow{g_*} H_{d,s}(\bar{C}) \xrightarrow{\delta} H_{d+1,s+1}(C') \longrightarrow \cdots$$

$f, g : C \to C'$ chain maps $(0,0)$

$f \simeq g$: if $\exists\, P : C \to C'$ of big $(1,0)$ st

$$f - g = \partial' \circ P + P \circ \partial .$$

Key prop : $f \simeq g \Rightarrow f_* = g_* : H(C) \to H(C')$.

Lemma. $V_i \bullet \simeq V_j \bullet : CG^- \to CG^-$

(that's why HG^- is a $\mathbb{F}[U]$-module).

For $\wedge$, CG^- / V_i $\mathbb{F}$

$W = \mathbb{F}_{(0,0)} \oplus \mathbb{F}_{(-1,-1)}$.

M bigr., $- \otimes W$ adds a copy of M w/ shifted sides.

$$M \otimes W \cong M \oplus M[\![1,1]\!]$$

$$M[\![1,1]\!]_{d,s} = M_{d+1,\, s+1} .$$

Thm. $\widetilde{GH}(\mathfrak{g}) \cong \underbrace{\widehat{GH}(\mathfrak{g}) \otimes W^{\otimes(n-1)}}$

Pf.
$$H\left(\frac{CG^-}{V_1=\dots=V_{j}}\right) \cong H\left(\frac{CG^-}{V_1=0}\right) \otimes \underline{W^{\otimes(j-1)}}$$
$j=1,\dots,n$

Induction: $\underline{j=1}$. ✓

$\underline{j>1}$.
$$0 \to \frac{GC^-}{V_1=\dots=V_{j-1}=0} \xrightarrow{V_1\cdot} \frac{GC^-}{V_1=\dots=V_{j-1}=0} \xrightarrow{\pi} \frac{GC^-}{V_1=\dots=V_j=0} \to 0.$$
$\wr\wr$

O_{wep}

les
$\rightsquigarrow$
$$0 \to \underbrace{H\left(\frac{CG^-}{V_1=V_{j-1}=0}\right)}_{(\Delta)} \xrightarrow[(0,0)]{\pi_*} \underbrace{H\left(\frac{GC^-}{V_1\dots V_j=0}\right)}_{(*)} \xrightarrow[\delta\,\mathrm{id}\ (1,1)]{\delta} \underbrace{H\left(\frac{GC^-}{V_1=V_{j-1}=0}\right)}_{(\Delta)}$$

$$(*)_j \simeq (\Delta)_j \oplus (\Delta)_{j-1}[1,1] \cong (\Delta) \otimes W$$

$$\overset{\text{induct}}{\simeq} \widehat{GH}(G) \otimes W^{(j-1)}$$

$\square$