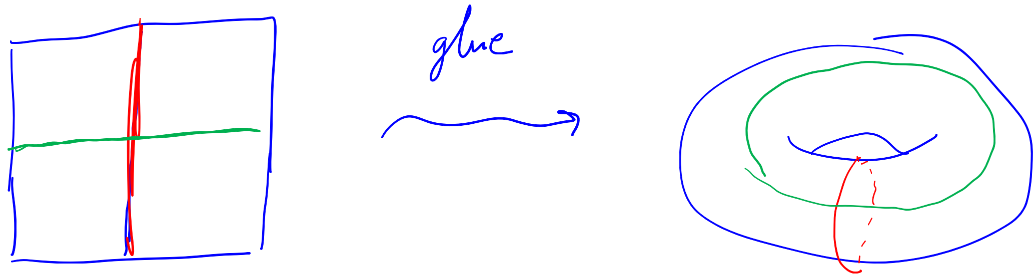
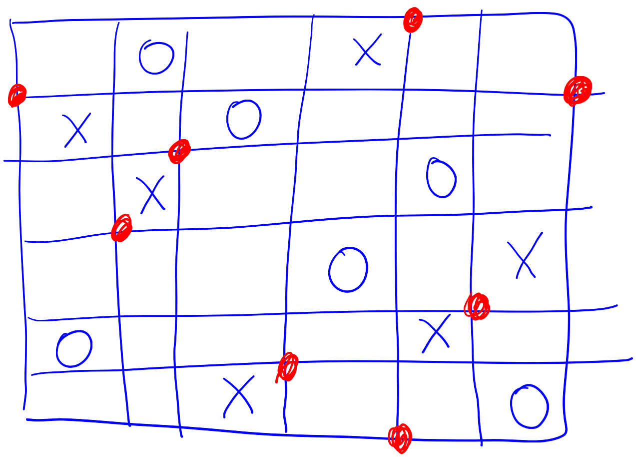
Grid states & rectangles

$\mathbb{G}$ toroidal grid diagram



Def: A grid state of $\mathbb{G}$ $(n \times n)$ is a bijection between horizontal & vertical circles. I.e., a collection of n pts $\mathbf{x} = \{x_1, \dots, x_n\}$ st every horizontal & vertical circle contains exactly one x_i.



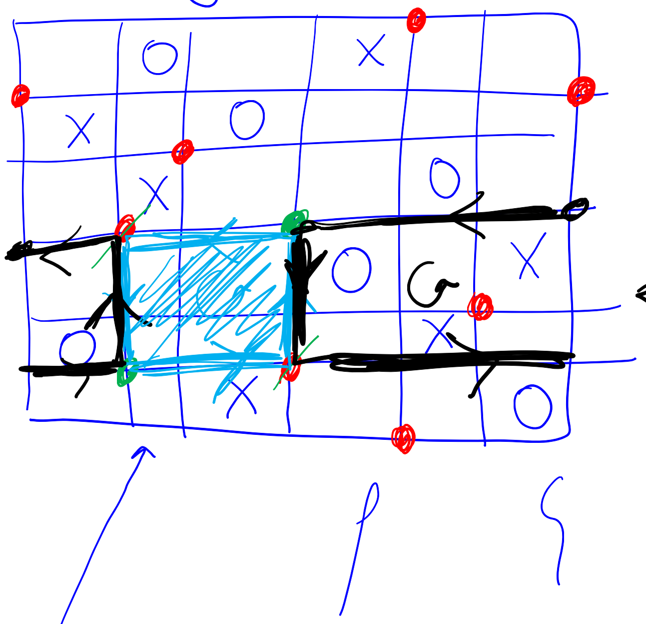
$S(\mathbb{G}) = \{$grid states for $\mathbb{G}\}$.

Now suppose $\mathbf{x}, \mathbf{y} \in S(\mathbb{G})$ st $\mathbf{x}, \mathbf{y} \in S(\mathbb{G})$ st $\#(\mathbf{x} \cap \mathbf{y}) = n-2$. Suppose that these 4 pts are the corners of a rectangle r in $\mathbb{G}$.

$y =$ •

$x =$ •

r



r is a rect. from y to x

horiz. green points to red

$:= r$ is a rect. from x to y

Q: $\# S(\mathbb{G})$?

$\mathrm{Rect}(x,y)$ = rectangles from x to y

Bigrading on $S(\mathbb{G})$

$$S(\mathbb{G}) = \coprod_{d,s} S_d(\mathbb{G}, s)$$

every x has a bidegree (d, s)

Maslov grading

Alexander grading

why?

$$\mathbb{Z}/2[S(\mathcal{G})] \cong \bigoplus_{d,s} \mathbb{Z}/2[S_d(\mathcal{G},s)].$$

relevant later!

<u>Prop</u>: Let $\mathcal{G}$ be toroidal. There is a function

$$M_O : S(\mathcal{G}) \longrightarrow \mathbb{Z}$$
$$x \longmapsto d$$

charact. by

(1) Write x^{NWO} for the grid state w/ $\boxed{O}$. Then

$$M_O(x^{NWO}) = 0.$$

(2) If $r \in \mathrm{Rect}(x,y)$ then

$$\boxed{\underline{M_O(x) - M_O(y)} = 1 - 2\#(r \cap \mathbb{O}) + 2\cdot\#(x \cap \mathring{r})}$$

Def: $$A : S(\mathcal{G}) \longrightarrow \tfrac{1}{2}\mathbb{Z}$$
$$x \longmapsto s$$

$$A(x) = \frac{1}{2}\left(M_{\mathbb{O}}(x) - M_{X}(x)\right) - \binom{n-1}{2}.$$

Prop:

1) A takes values in $\mathbb{Z}$.

2) $A(x) - A(y) = \#\left(r \cap X\right) - \#\left(r \cap \mathbb{O}\right)$. $\quad r \in \mathrm{Rect}(x,y)$