Cubical Singular Homology , Cubes vs tetrahedra

* Chains in X lin combinations of maps

$\sigma : I^k \longrightarrow X$; $\rightsquigarrow C_k(X)$

$\partial : C_k(X) \to C_{k-1}(X)$

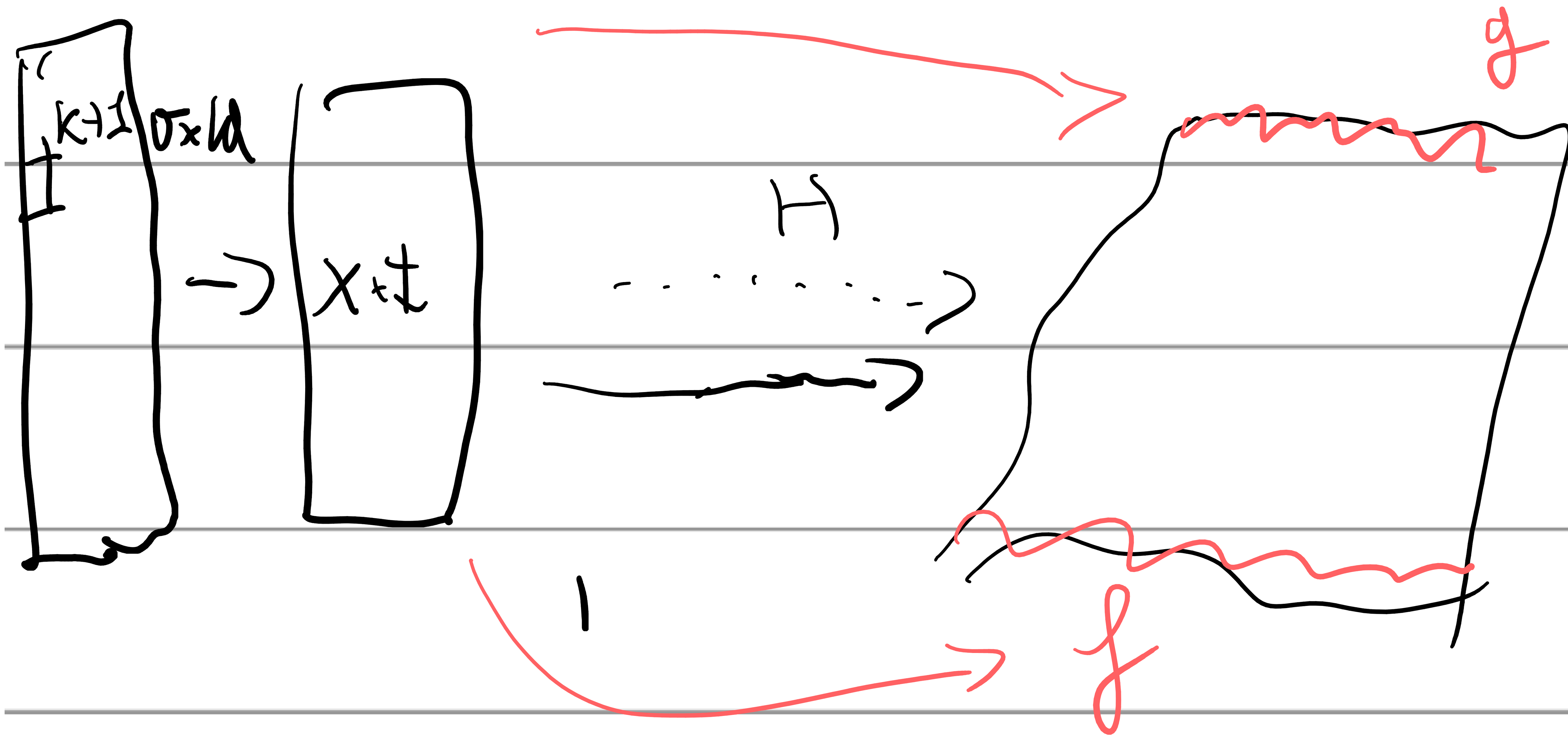
Given homotopic maps $f, g : X \to Y$

Why do we get a chain homotopy $C_\cdot(X)$ & $C_\cdot(Y)$?

$$\begin{tikzcd}
\cdots \arrow[r] & C_{k+1}(X) \arrow[r, "\partial"] \arrow[d, "f_*"', "g_*"] & C_k(X) \arrow[r] \arrow[d, "f_*"', "g_*"] \arrow[dl, "P"'] & \cdots \\
\cdots \arrow[r] & C_{k+1}(Y) \arrow[r, "\partial"] & C_{k+1}(Y) \arrow[r] & \cdots
\end{tikzcd}$$

$H : X \times I^1 \longrightarrow Y$ s.t. $H|_{X\times\{0\}} = f$; $H|_{X\times\{1\}} = g$

Recall $f_*(\sigma) := f \circ \sigma$, $f \circ \sigma : I^k \longrightarrow X \longrightarrow Y$



(X and σ in between)

$$\underset{}{P(\overset{\text{k-chain}}{\sigma})} = H \circ (\sigma \times \mathrm{Id}_I)$$

$$\partial P + P\partial = f - g \quad \text{or} \quad \partial \overset{\sigma}{P} = \overset{\sigma}{f} - \overset{\sigma}{g} + P\overset{\sigma}{\partial}$$

Oscar: Long exact seq:

$$\to KH^{i,j}(\,)(\,) \to KH^{\cdot\cdot}(\times) \to KH^{\cdot\cdot}(\asymp)$$

$$\downarrow$$

$$KH(\,)(\,) \to$$

$c := \# n_- - \# n_+$

$V = \mathbb{Q}[X]/X^2$

$$V_\alpha = V^{\otimes \#\text{ circles}} \{|\alpha| + n_+ - 2n_-\}$$

$$C^{i,*}(D) = \bigoplus_{\substack{\alpha \\ |\alpha| = i + n_-}} V_\alpha$$

For $\bigcirc$, $\#$ circles $= 1$, $n_+ = n_- = |\alpha| = 0$

$$0 \to 0 \to \underline{V} \to 0 \to 0 \to \cdots$$

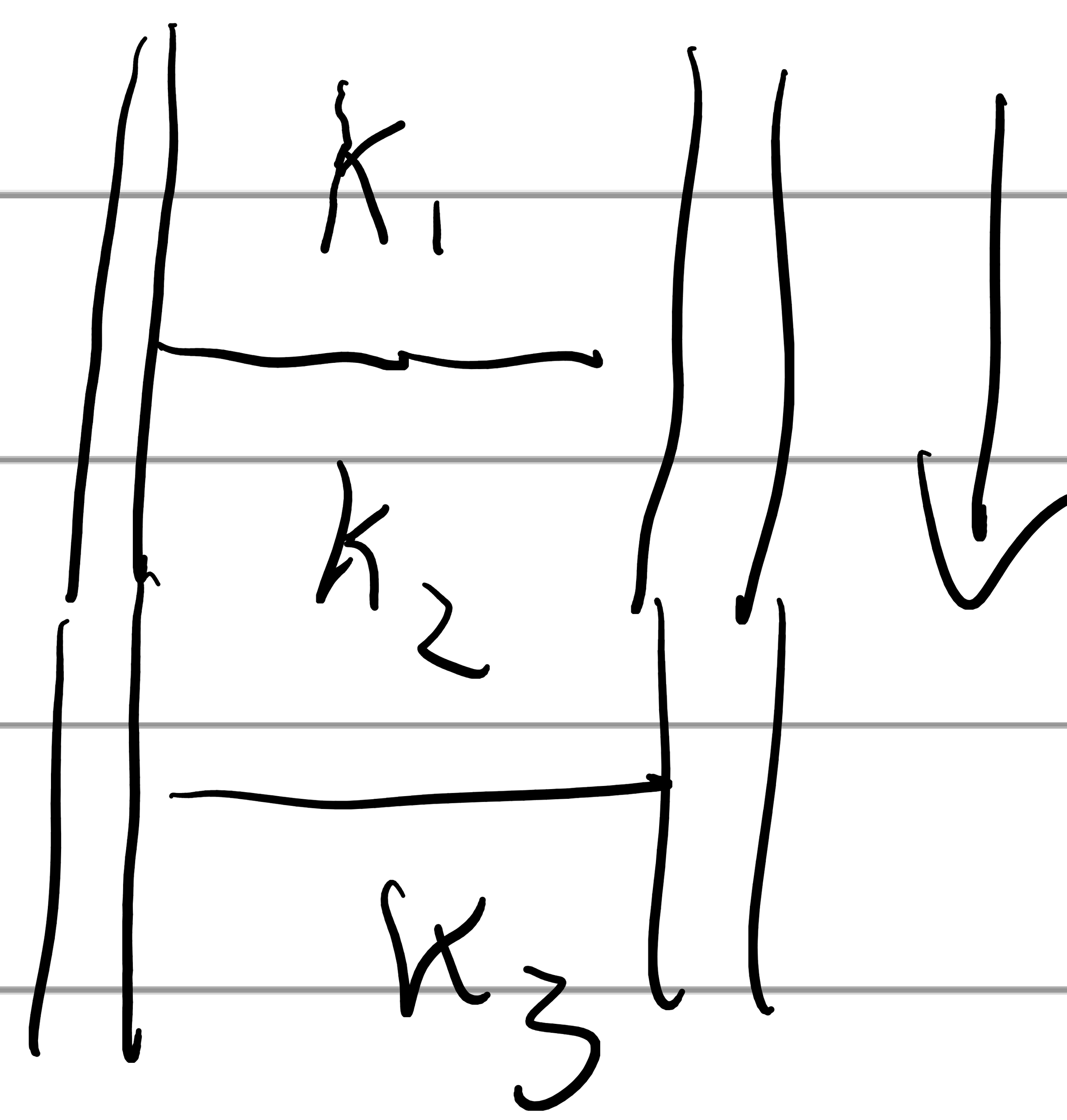
Goals: Movies

Cobordism surfaces $\Sigma \subset \mathbb{R}^3 \times I$

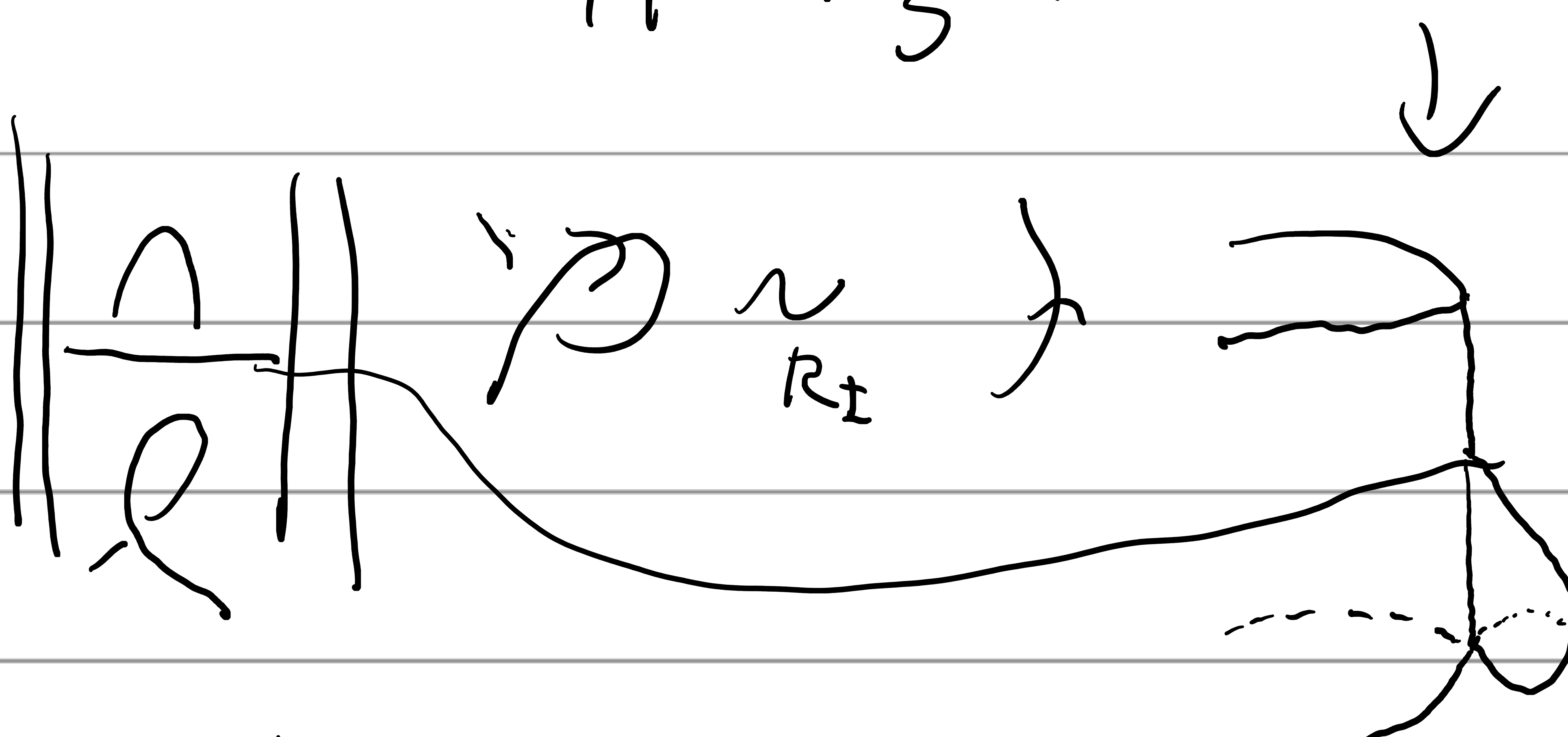
$\partial \Sigma = \overline{L}_0 \sqcup L_1 \qquad L_i = \mathbb{R} \times \{i\}$

$i \in \{0,1\}$

rep by movies

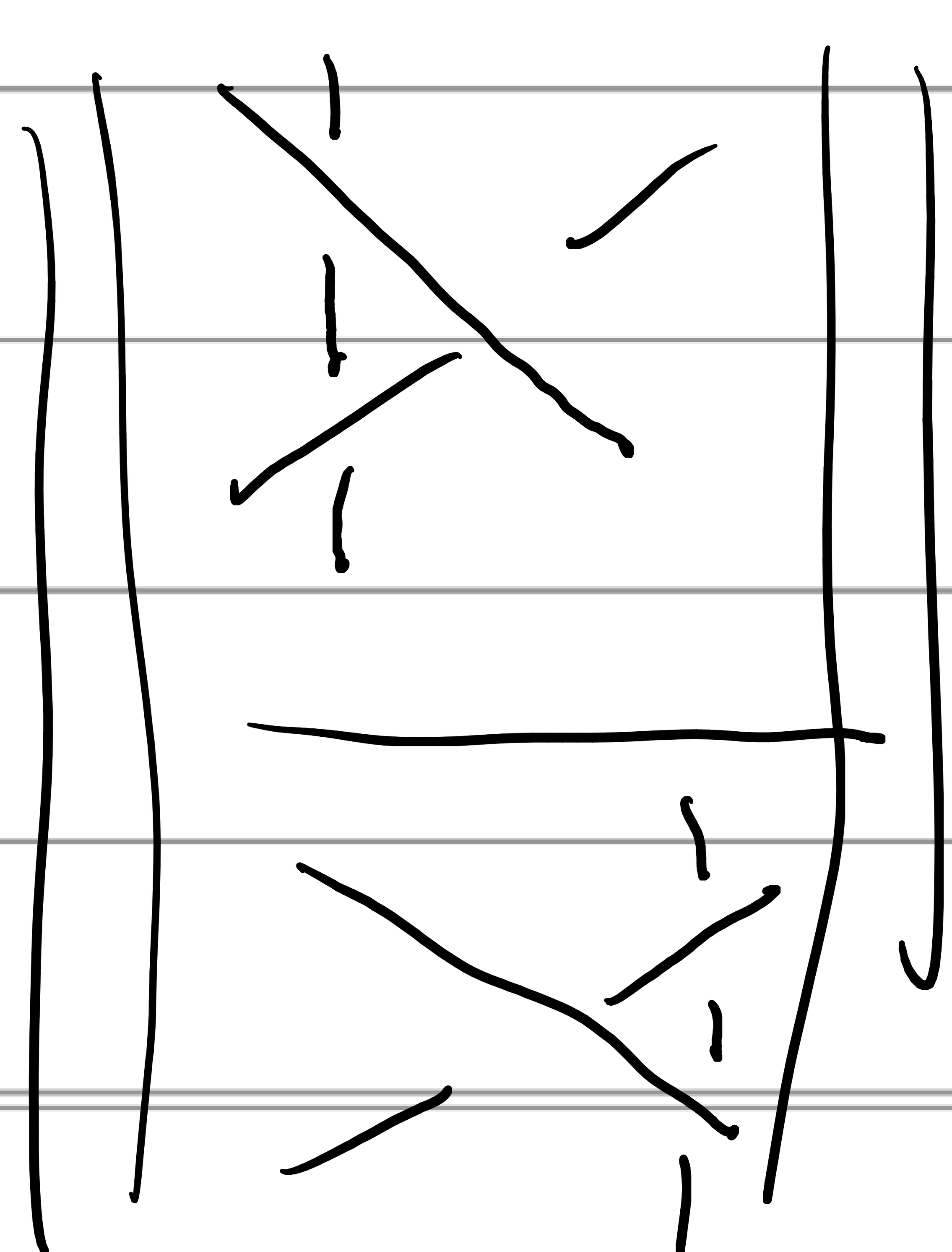


R_I



R_{II}

, R_{III}:

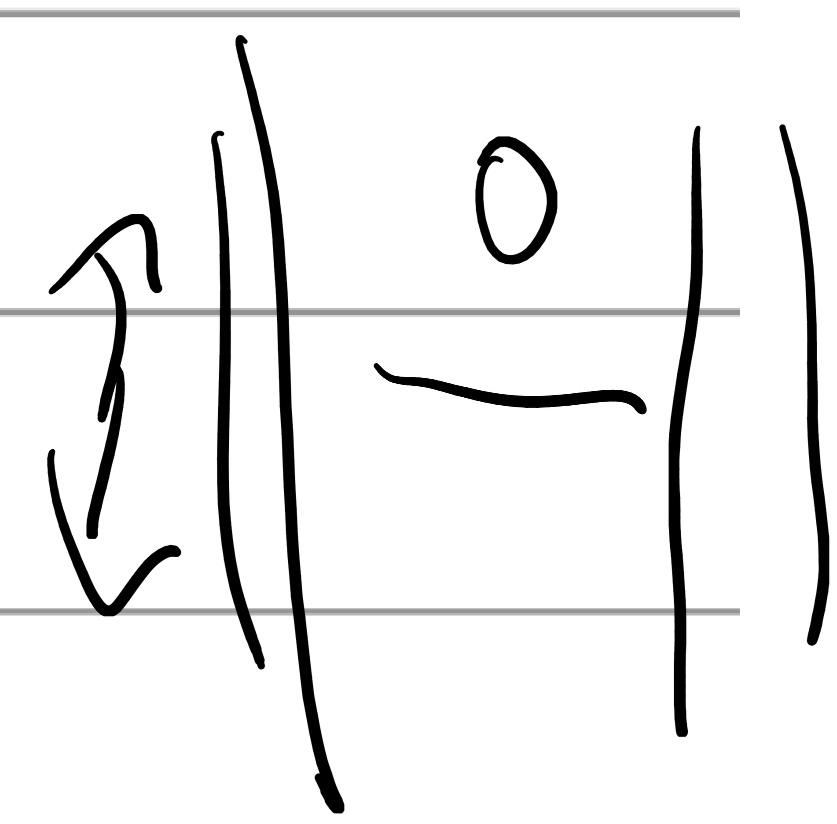


What if we add or delete the unknot

$$C^{\bullet,\bullet}(D \sqcup O) = C^{\bullet,\bullet}(D) \otimes V$$

since the # circles $\to +1$

Cobordism



$$KH(D) \to KH(D) \otimes V$$

$$v \to v \otimes \eta \qquad \mathrm{id} \otimes \eta$$

$$\eta(1) = 1_v \quad \deg 1, \; x$$

$$KH^{i,j}(D) \to KH^{i,j+1}(D \sqcup O)$$

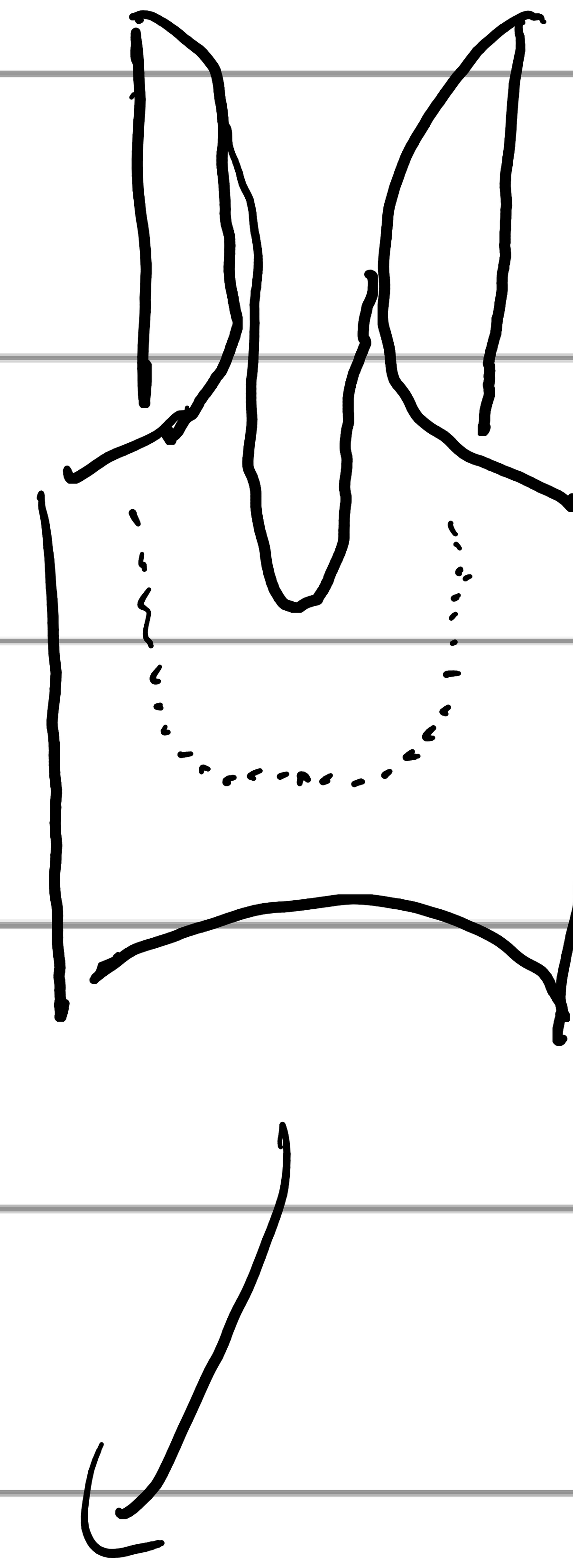
$1 \to 0$

$$\varepsilon(1_v) = 0, \qquad \varepsilon(X) = 1_{\mathbb{Q}}$$

degree -1 → degree "0"

$$KH^{i,j}(D \sqcup O) \to KH^{i,j+1}(D)$$

$$KH^{i,j}(D \pm O) \to KH^{i,j+1}(D)$$



μ

Δ

$C^i \in V^{\otimes \cdot}\{\cdots\}$

	input		image	degree
μ	$1 \otimes 1$	$\to$	1	$2 \to 1$
	$1 \otimes x,\ x \otimes 1$	$\to$	x	$1-1=0 \to -1$
	$x \otimes x$	$\to$	0	$-2 \to 0$
Δ:	x	$\to$	$x \otimes x$	$-1 \to -2$
	1	$\to$	$1 \otimes x + x \otimes 1$	$1 \to 0$

$\boxed{KH(\,)(\,)} \to KH(\asymp)\{1\}$