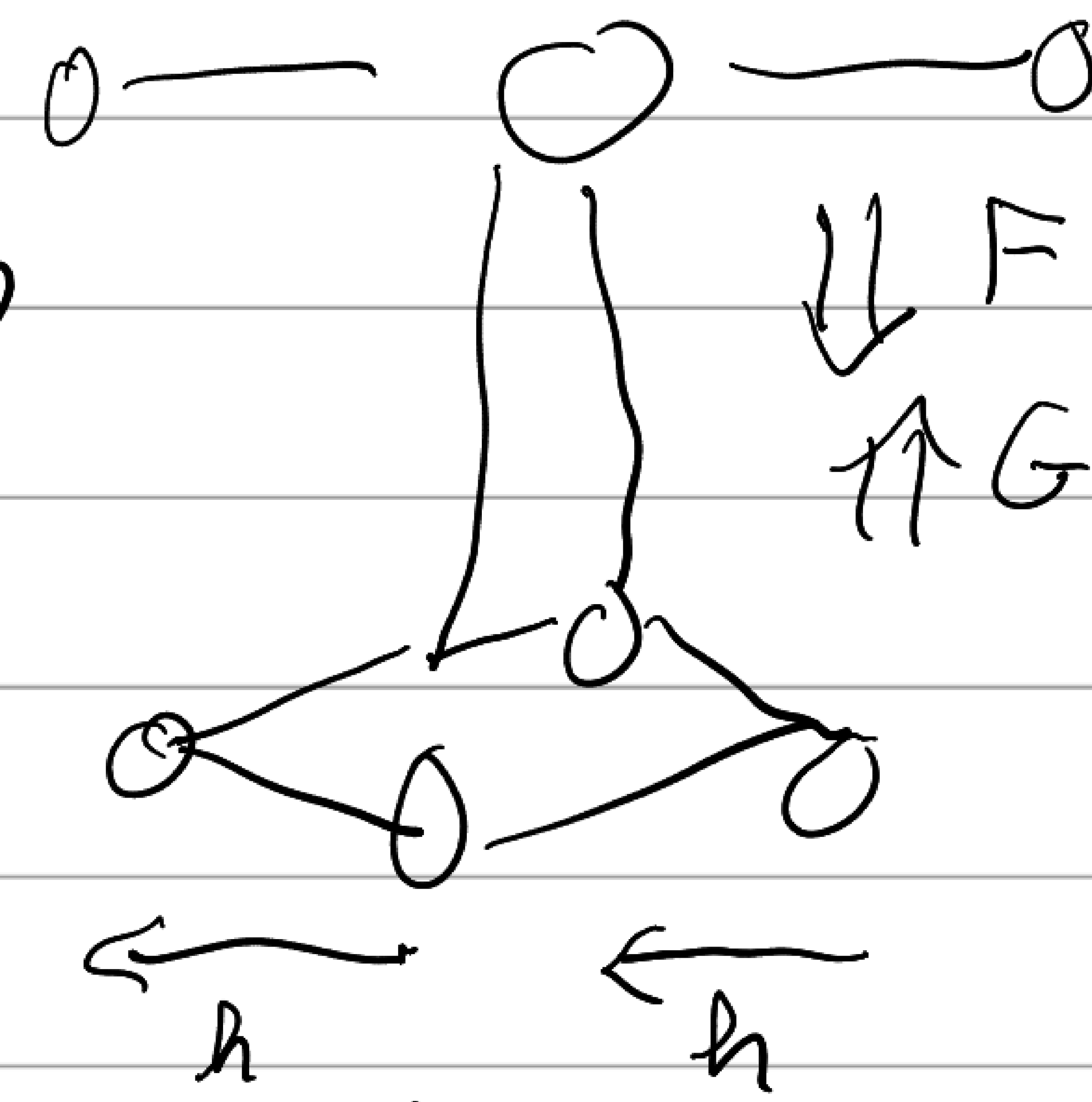
Revisiting Aarnoat

Picture, JpG



- $dF = 0$
- $Gd = 0$

$\Big\} \to$ complex morphisms

- $G \cdot F = \mathrm{id}$

$\bigcirc = 0$

- $FG - 1 = hd + dh$

Def $\Omega \underset{F}{\overset{G}{\rightleftarrows}} \Xi$, then G is a "deformation retraction" of F if

- $h : \Omega_a \to \Omega_{a-1}$ s.t. $hF = 0$
- $GF = \mathrm{id}$
- $\mathrm{id} - FG = d_\Omega h + h d_\Omega$

$hF = 0$, $\mathrm{id} - GF \neq 0$

$\Psi : \Omega \to \Xi$, $d : \Omega$, $\delta\ \Xi$

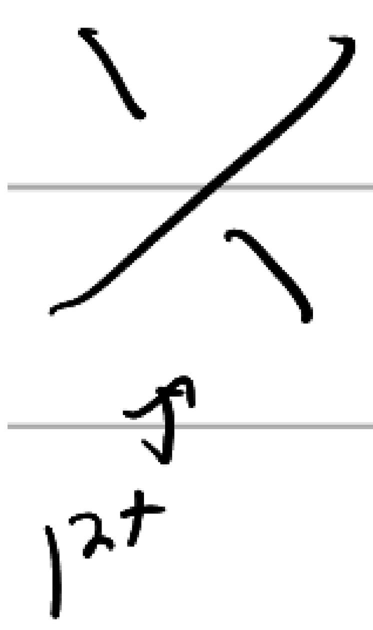
def "cone" $\Gamma(\Psi)$

$$\Gamma(\Psi)^a = \Omega^{a+1} \oplus \Xi^a$$

$$d_\Gamma := \begin{pmatrix} -d^{a+1} & 0 \\ \psi^{a+1} & \delta^a \end{pmatrix} = \begin{pmatrix} -d^{+1} & 0 \\ \psi^{+1} & \delta \end{pmatrix}$$

$$\begin{tikzcd}
\Omega^a \arrow[r, "-d"] \arrow[d, "\psi^a"] & \Omega^{a+1} \arrow[r, "-d^{+1}"] \arrow[d, "\psi^{a+1}"] & \Omega^{a+2} \arrow[d, "\psi^{a+2}"] \\
\Xi^a \arrow[r, "\delta"'] \arrow[ur, phantom, "\oplus"] & \Xi^{a+1} \arrow[r, "\delta^{+1}"'] \arrow[ur, phantom, "\oplus"] & \Xi^{a+2}
\end{tikzcd}$$

$$d_F^2 = \begin{pmatrix} d^2 & 0 \\ -\psi^{+2} d^{+1} + \delta^{+1} \psi^{+1} & \delta^2 \end{pmatrix} = 0$$

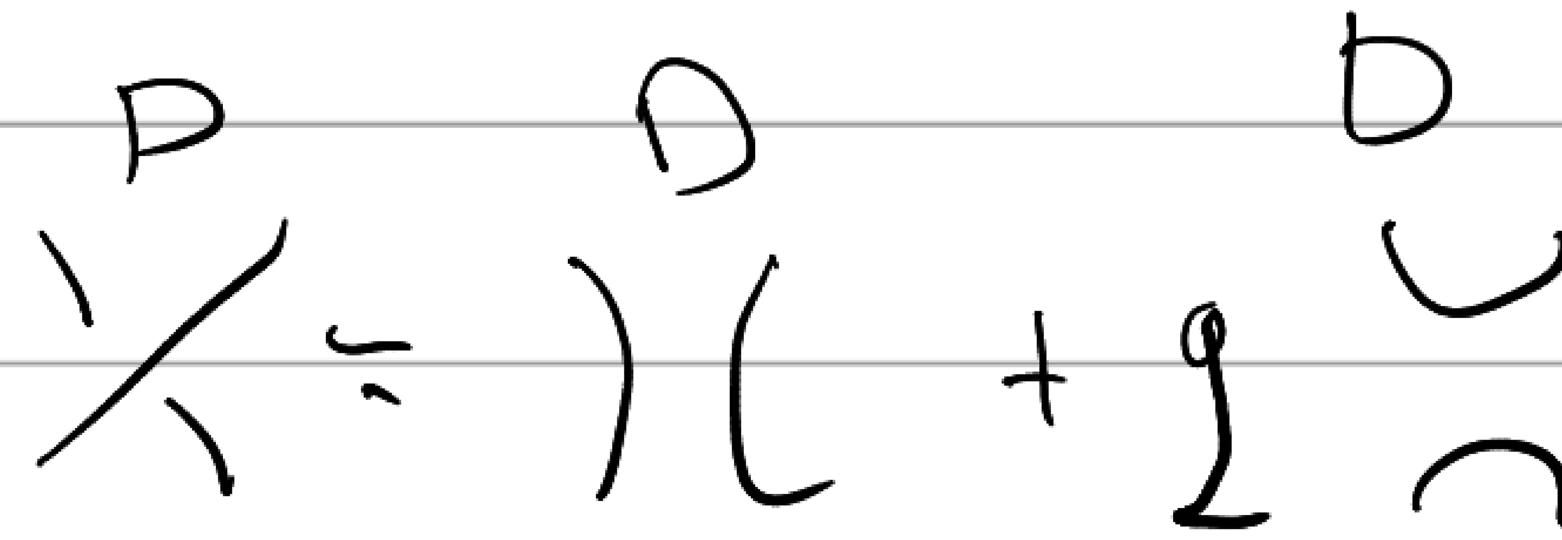


$D \simeq \oplus V_\alpha = \oplus V_{0\alpha} \oplus V_{1\beta}$

$\simeq D_0 \oplus \underline{q} D_1$

$= \Gamma(\psi)[-1]$

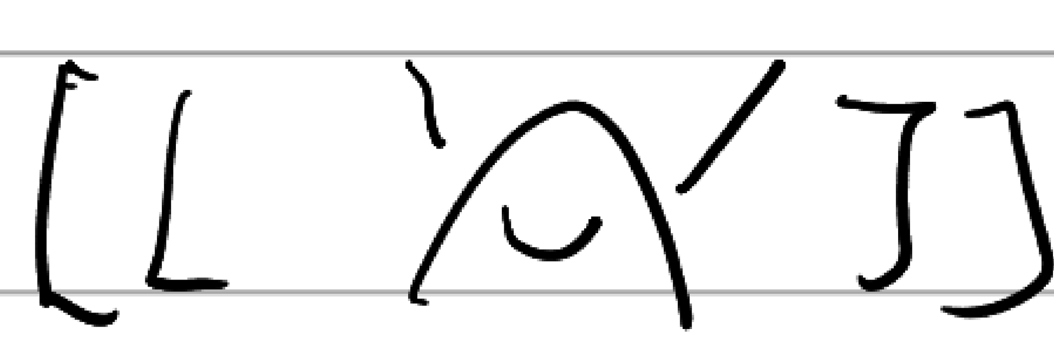
$\psi =$ 



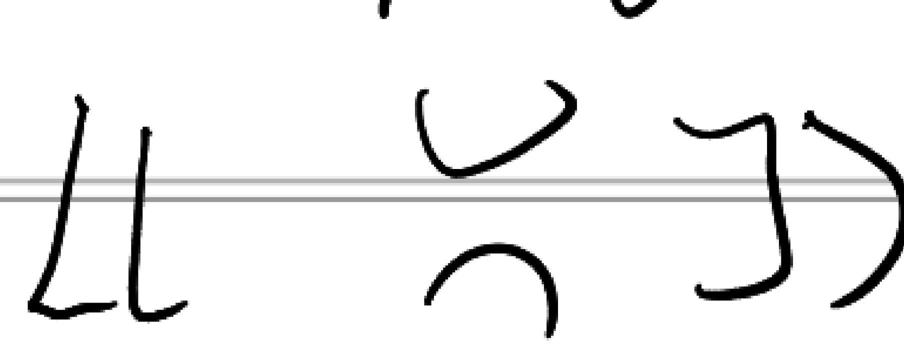
Lemma: $\Psi : \Omega \to \Xi$

$$\Gamma(\Psi F) \simeq \Gamma(\Psi) \simeq \Gamma(F'\Psi)$$

$$\begin{tikzcd}
h \quad \Omega \arrow[r, "\Psi"] \arrow[d, shift right, "G"'] & \Xi \quad h \arrow[d, shift right, "F'"'] \\
\Omega' \arrow[u, shift right, "F"'] & \Xi' \arrow[u, shift right, "G'"']
\end{tikzcd}$$

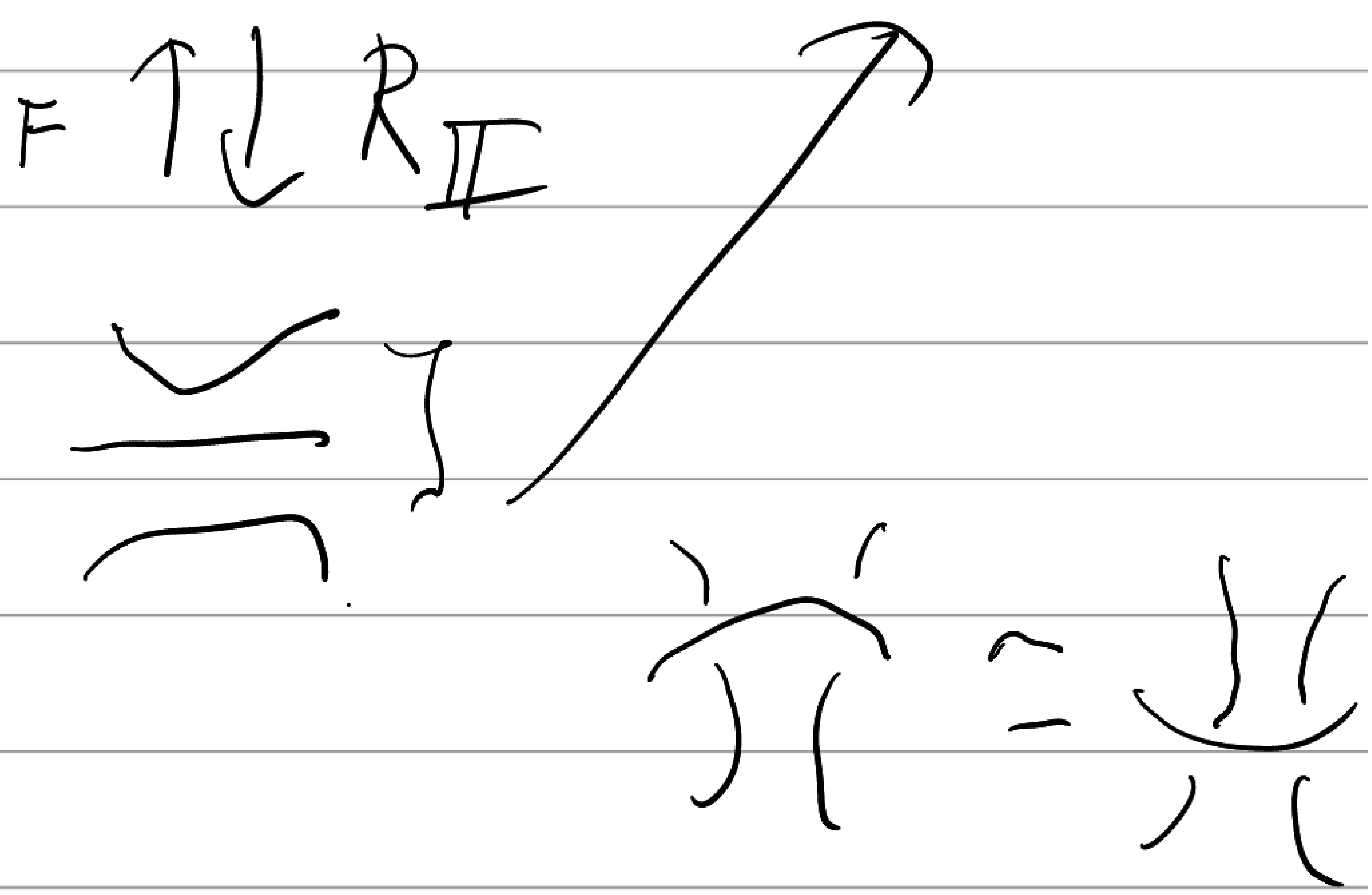
e.g. 

$G \uparrow \downarrow F$ $\qquad R_{II}$



R_{III}:  R_{III}

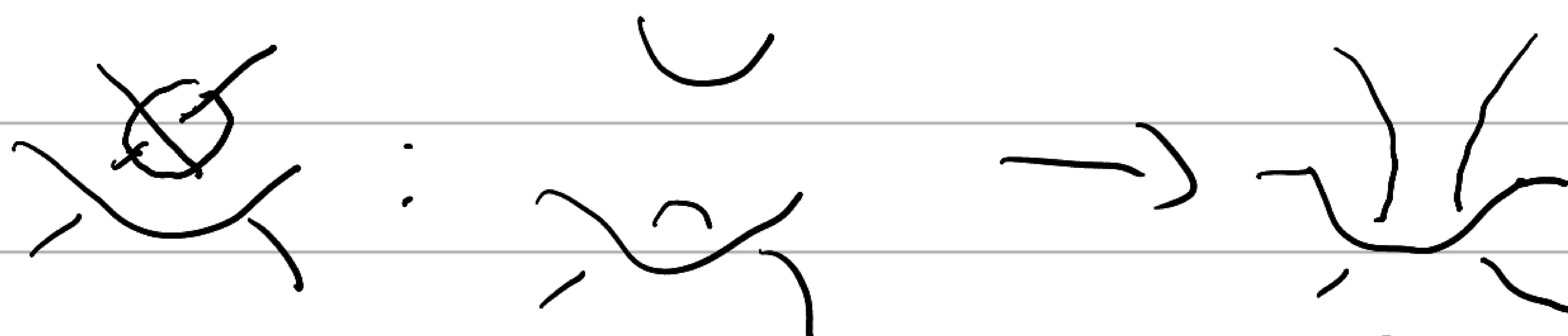
Ψ: 

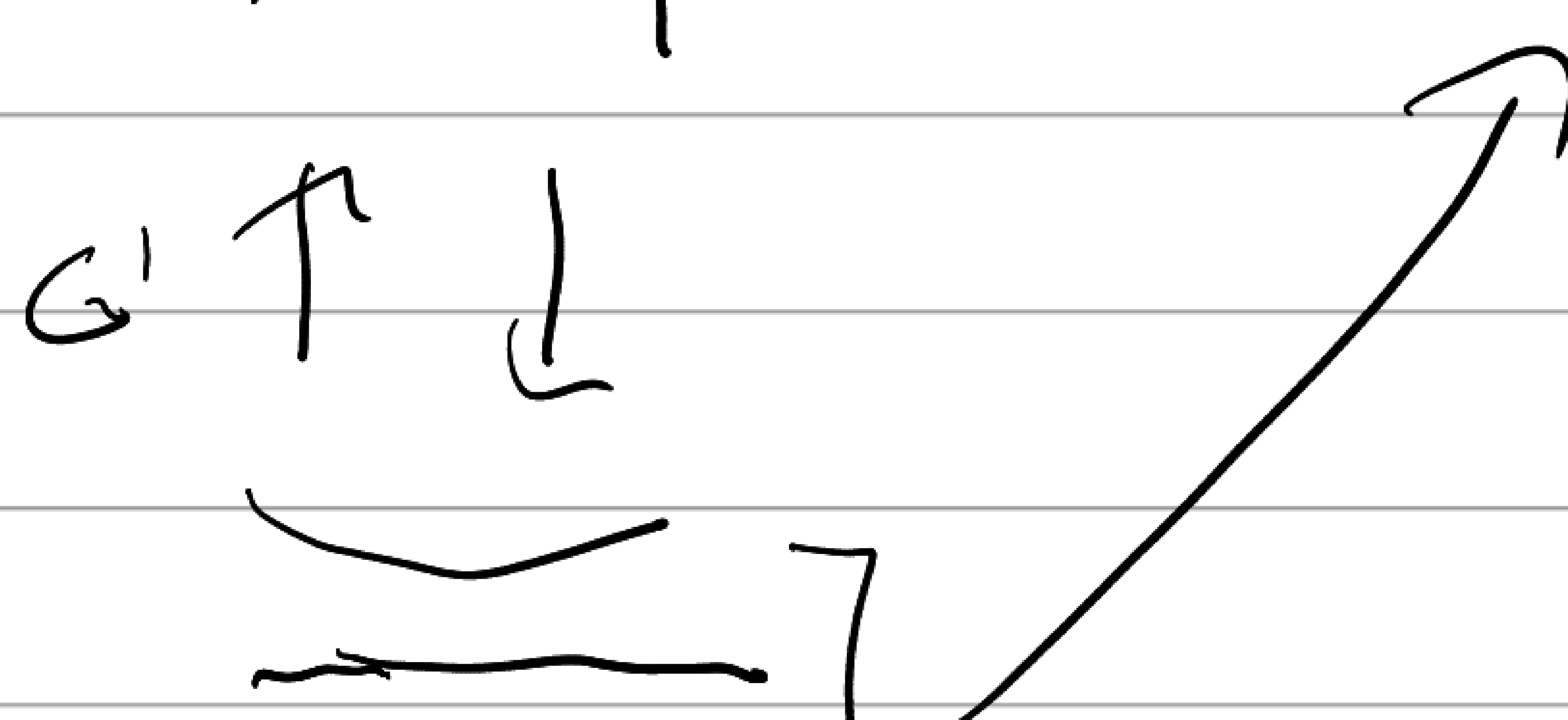


$\Gamma(\Psi)$

$\cong$

$\Gamma(\Psi F)$

Φ: 



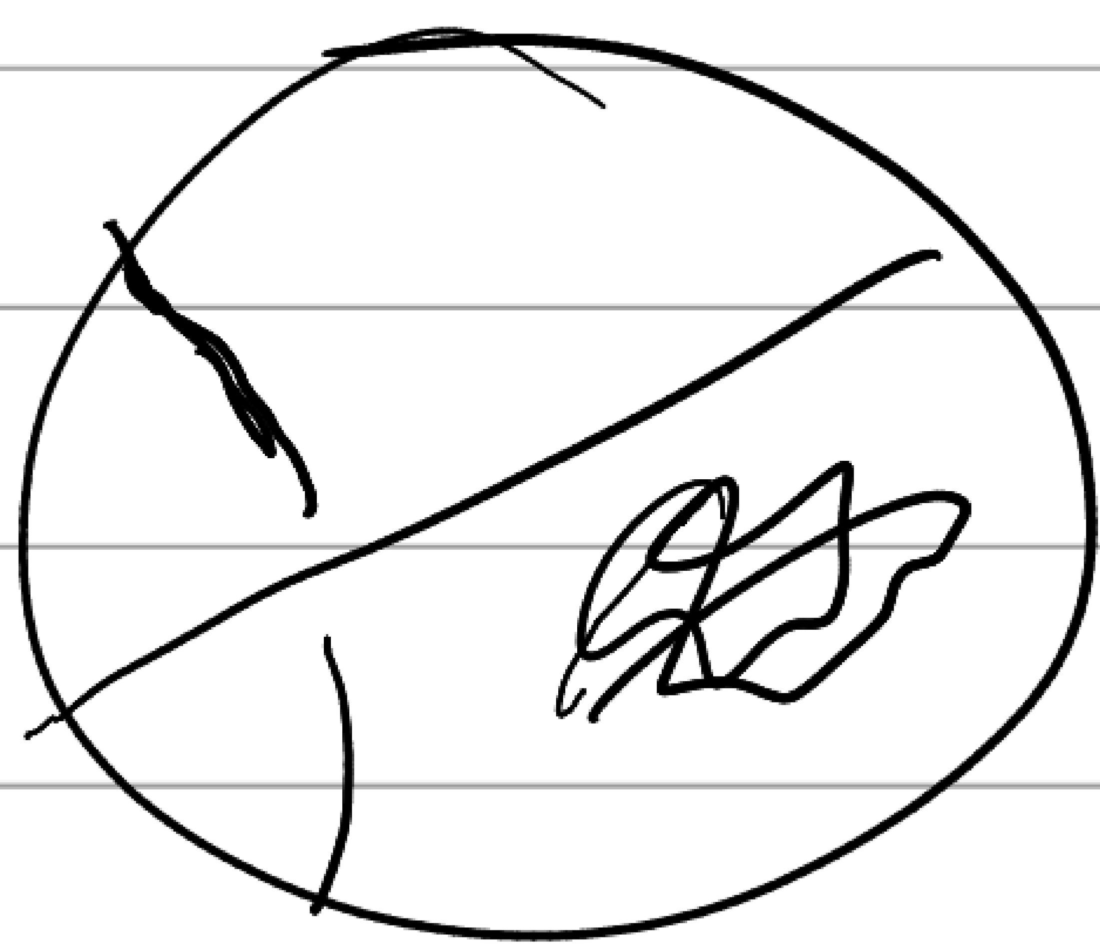
$\Gamma(\Phi)$

$\cong$

$\Gamma(\Phi G')$

$\Gamma(\Phi G') \simeq$
$\Gamma(\Psi F)$

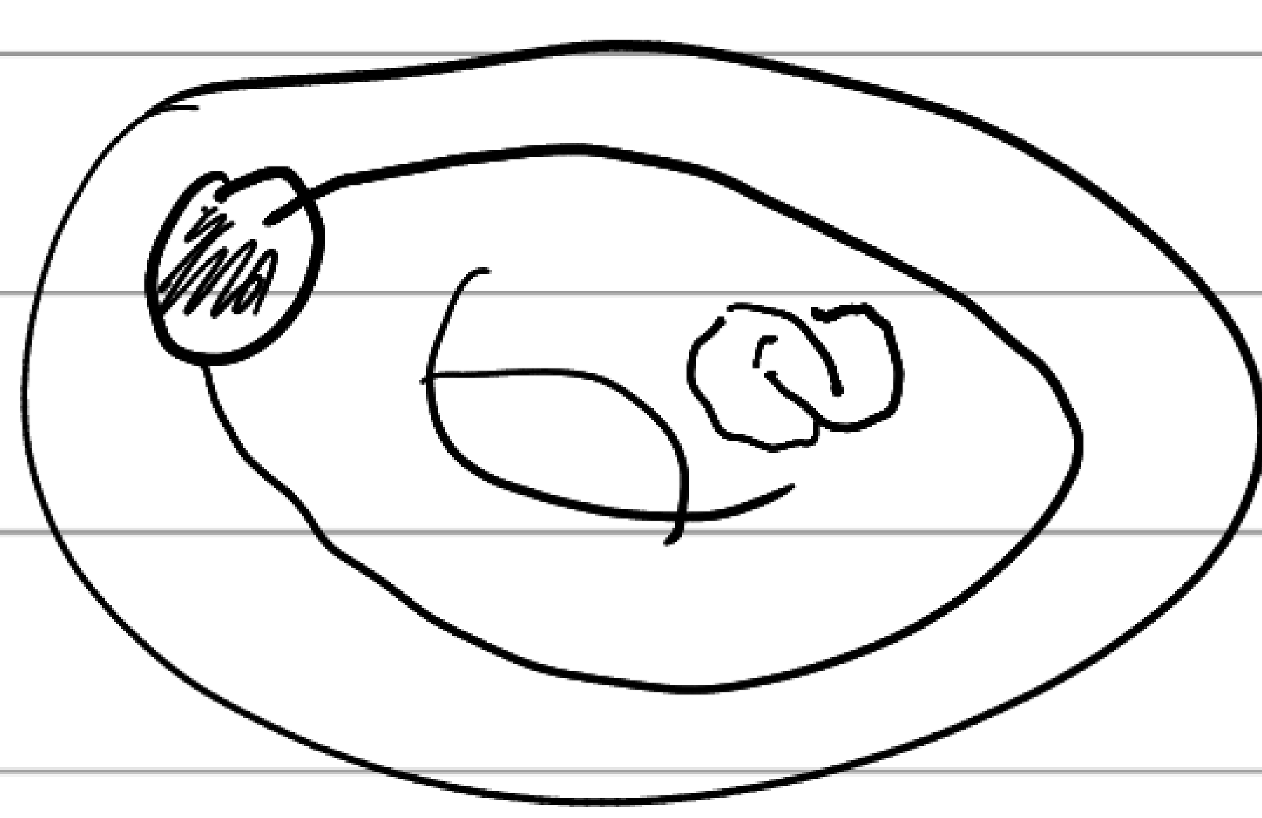
tangles are arcs in discs



$t^0(k)$ = isotopy class with k ends

$t(k)$ mod Reidemeister

Closed surfaces with boundaries
$S_g \setminus$ open discs

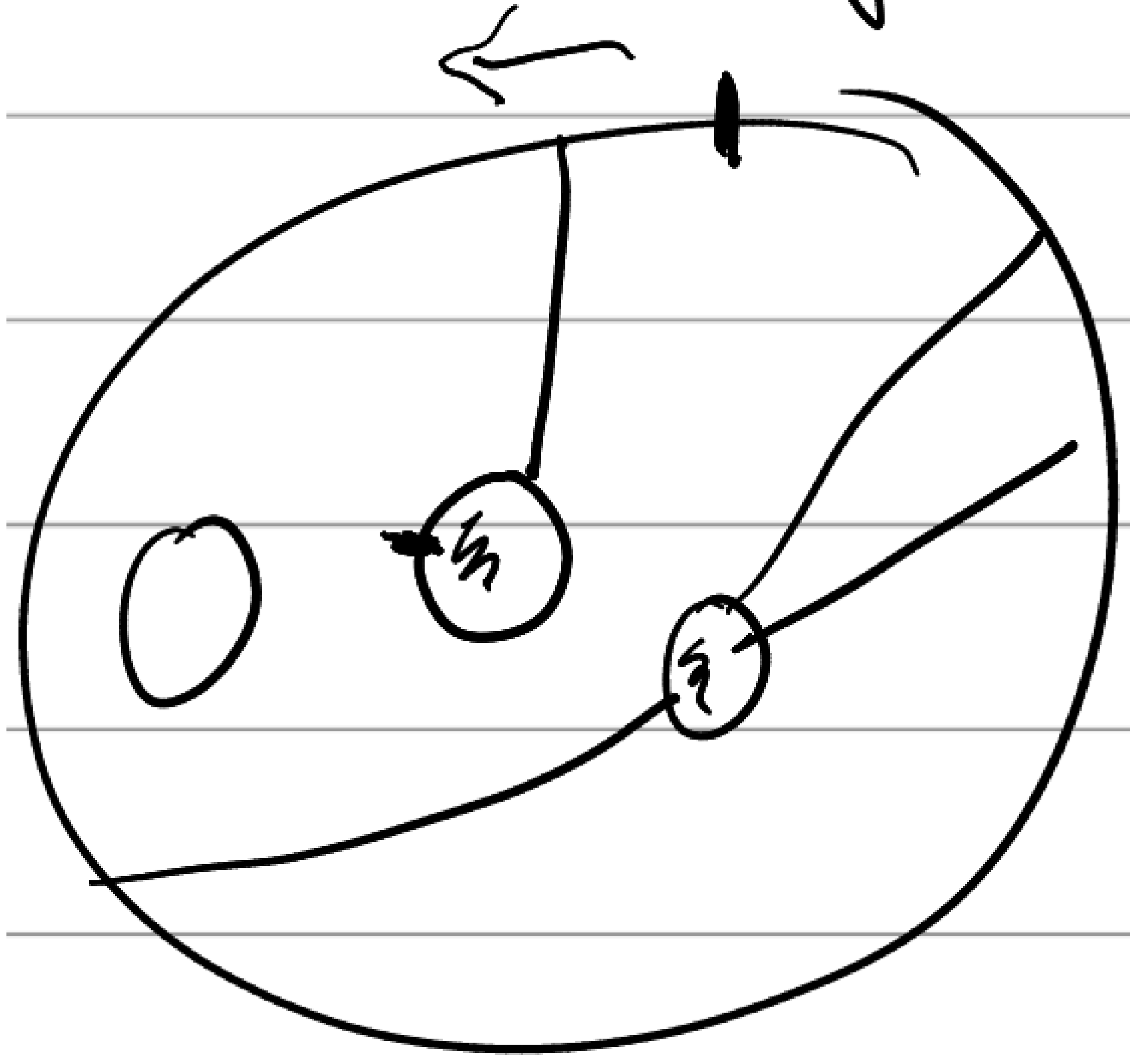


$t(k, g, m)$

g → genus

m → # discs removed

Def. planar arc diagram
- a disc with d internal discs removed
- on each of the $d+1$ discs "marking"
- non-intersecting arcs / mod isotopies



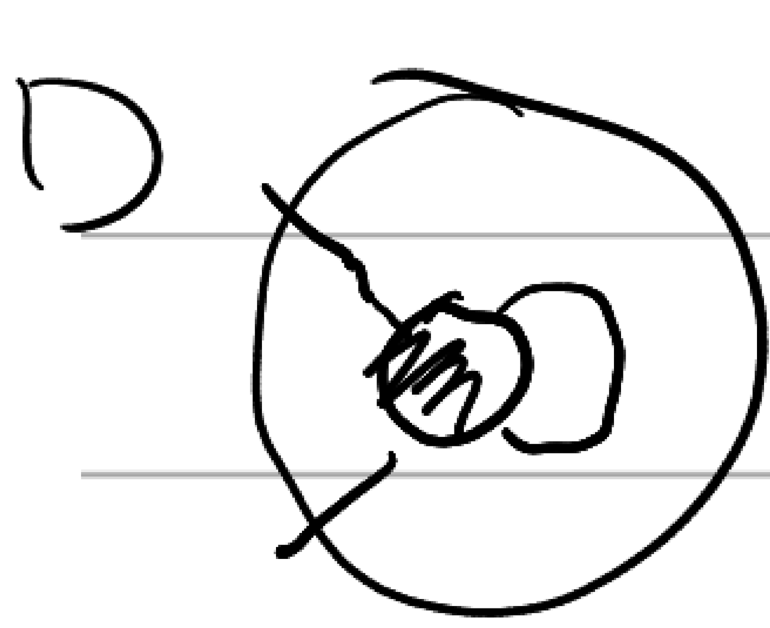
D d discs removed

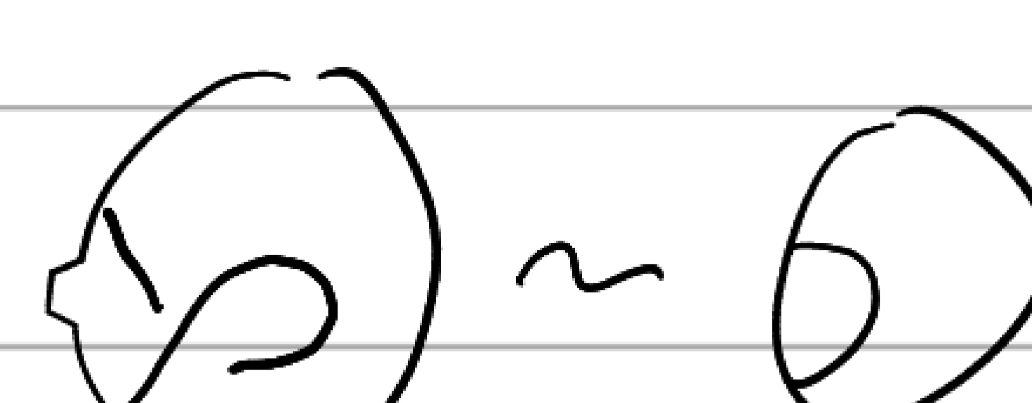
$k_1 \dots k_d$, k_{d+1}

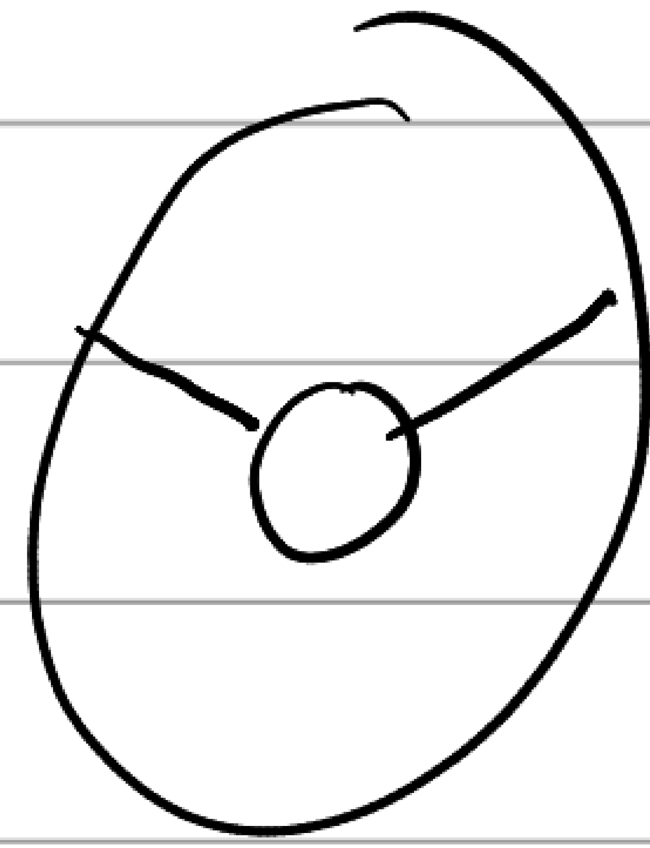
$$D: T^0(k_1) \times \dots \times T^0(k_d) \longrightarrow T^0(k_{d+1})$$

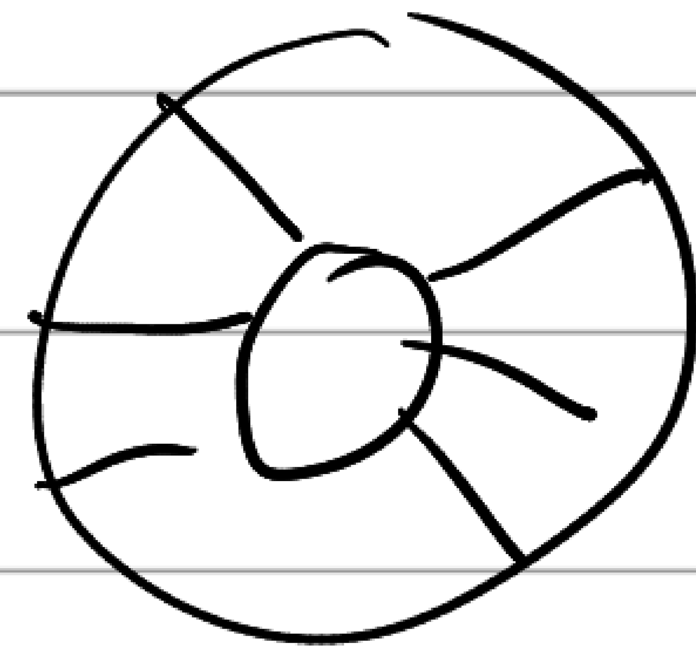
Reidemeister $\hookrightarrow$ Reidemeister external lines

$$D: T(k_1) \times \dots \times T(k_d) \longrightarrow T(k_{d+1})$$



$T(2) =$  

D  $\mathcal{T}(2) \to \mathcal{T}(2)$

 $\mathcal{T}(6) \to \mathcal{T}(6)$

Planar algebras: "$P(K)$

planar diagrams ⇝ planar diagram maps

$D \rightsquigarrow D_P : P(K_1) \times \dots \times P(K_d) \to P(K_{d+1})$

$P(K) = \mathcal{T}(K) = \mathcal{O}bj(\mathcal{C}ob_{/\ell}(K))$

$P(K) = \mathcal{M}or(\mathcal{C}ob(K))$ $\quad D \times I$

$\mathcal{M}at(\mathcal{M}or(\mathcal{C}ob)))$

$\mathcal{K}om(\mathcal{M}at(\mathcal{M}or(\mathcal{C}ob)))$

$\{\Omega_i, d_i\}_{i \in \mathbb{Z}}$

$$\Omega_D^r := \sum_{r = r_1 + \dots} D(\Omega_1^{r_1}, \dots, \Omega_s^{r_s})$$

$$d_D = \sum_{i=1,\dots,s} (-1)^{(\dots)} D(I_{\Omega_1}, \dots, d_i, \dots I_{\Omega_s})$$

$$d_D^2 = \quad i \gamma \quad -\gamma i \quad , \quad i^2 \quad d_i^2 = 0$$

$$\psi : \Omega_i \to \Omega_i'$$

$$D(I, \dots, \psi, \dots) : D(\Omega_1, \dots)$$

$$\to D(\Omega_1, \dots, \Omega_i', \dots, \Omega_s)$$

$$\underline{\to C} \quad \Rightarrow \quad Ch(C)$$

$$T \rightsquigarrow D_T \qquad 1 \dots n$$

$$[[T]] = D_T([[X_1]], \dots, [[X_n]])$$